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Shape analysis. Genetic Algorithms for generic curves interpretation and analytical curves restitution.

The main goal of this research activity is 3D data analysis and interpretation to study the geometric shaping of architectural elements belonging to Cultural Heritage moving from Point Cloud Semantic Segmentation to 3D modelling. We are testing *Evolutionary Computing* (EC) to study these shapes merging critical human-driven interpretation and *Genetic Algorithms* (GAs) driven interpretation. GAs allow to automate identification of open or closed polycentric curves (e.g. ovals) and of analytic curves (e.g. ellipse) starting from reality based planar sections (irregular polylines) extracted from point clouds. GAs, based on natural selection, are tools for solving both constrained and unconstrained optimization problems not well suited for standard algorithms. Moving from theoretical knowledge and treaties study, the research topic is the shape analysis and restitution of hemispherical domes profile ('pointed arches' or polycentric arches), i.e. 'pointed domes', and of oval/ellipsoidal domes (generic and revolution surfaces). Starting from rules (from centers to points along profiles), our aim is to test a process to generate polycentric (closed or open) and elliptic profiles nearest to reality-based sections (from point along reality-based profiles to centers layout). The Evolutionary Solver (*Galapagos* component, *GH, Rhino*) starts from the identification of a given number of points (*GENE POOL, i.e.* variables) along the irregular reference polyline to select the best solutions (*SELECTION*) towards the optimal solutions (continuous semi-ideal profile) that best fit the reference curve by minimizing or maximizing different comparison conditions between the two curves (*FITNESS*).

Keywords:
Genetic Algorithms; Shape analysis; Analytic curves; Optimization; Cultural Heritage

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1. INTRODUCTION

The core of this research activity in Cultural Heritage is 3D data analysis and critical interpretation to find geometric layout used by designer to compose an architectural system. Research topic is the analysis of domes geometric nature that moves from theoretical knowledge and historical manuals study. The main goal is to define a process that can be used to study the geometric shaping of different elements and their parts, moving from Point Cloud Semantic Segmentation to 3D modelling. We are especially testing *EC-Evolutionary Computing* approach (*GAs-Genetic Algorithms*) that generates solutions to optimize problems using techniques inspired by natural evolution, such as inheritance, mutation, selection, and crossover to find 'semi-ideal' or *ideal analytic curves* that best fit and decode complex elements. The main goal is to generate a database of functions, based on geometric and mathematic rules using VPL (*Visual Programming Language*), to drive GAs for interpretation and restitution of generic polycentric profiles (open and closed) and analytic curves (*i.e.* conics, and in particular ellipse) starting from reality-based elements.
According to an interdisciplinary approach, EC involves urban and architectural design applications (Buffi et al., 2020; Canestrino et al., 2020, Palma et al., 2021), analytical-structural applications (Grillanda et al. 2017; Khan, 2015), manufacturing complex elements-systems (Zaremba, 2016; Coutinho, 2010; Limonge et al., 2010) and analytical-geometric applications for shapes restitution (Bianconi et al., 2018, Santagati et al., 2018). These approaches could be also used to support machine learning and AI processes (Sim, 2020). To complete the state of the art, in addition to GAs, it is necessary to mention the existence of other algorithms (Gatti, 2020). The EC is a methodological approach for solving both constrained and unconstrained optimization problems that is based on natural selection, the process that drives biological evolution. A GA repeatedly combines a population of individual solutions. At each step, it selects individuals at random from the current population to be parents and uses them to produce next generation. Over successive generations, the population "evolves" toward an optimal solution. It is possible to apply GAs to solve a variety of optimization problems that are not well suited for standard optimization algorithms, including problems in which the objective function is discontinuous, nondifferentiable, stochastic, or highly nonlinear. The solution depends on the combination of all the variables involved and the provided optimal alternatives are compared with specific design needs. Despite rapid technical and technological advances (ICT), AI do not satisfy qualitative data interpretation (Bianchini, 2014). However,

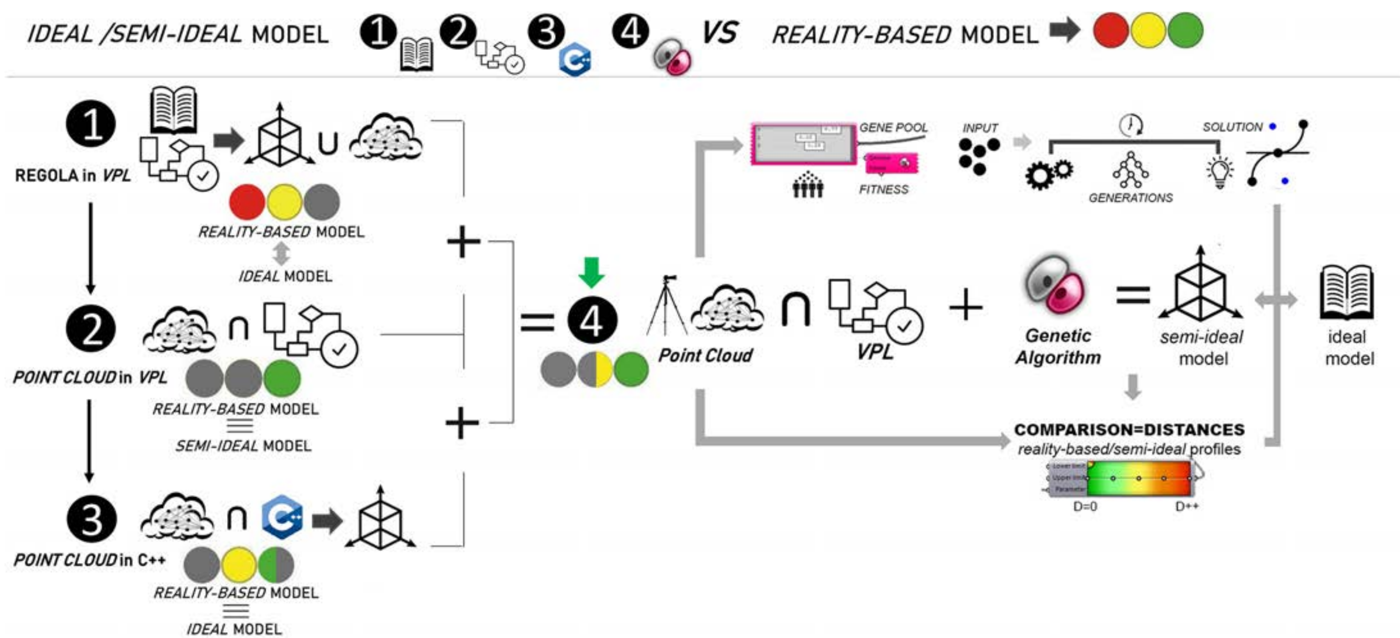

Fig. 1 – Methodological workflow: from traditional VPL procedural modelling (rules from Teatries) to semi-automatic Evolutionary Computing (Gas)

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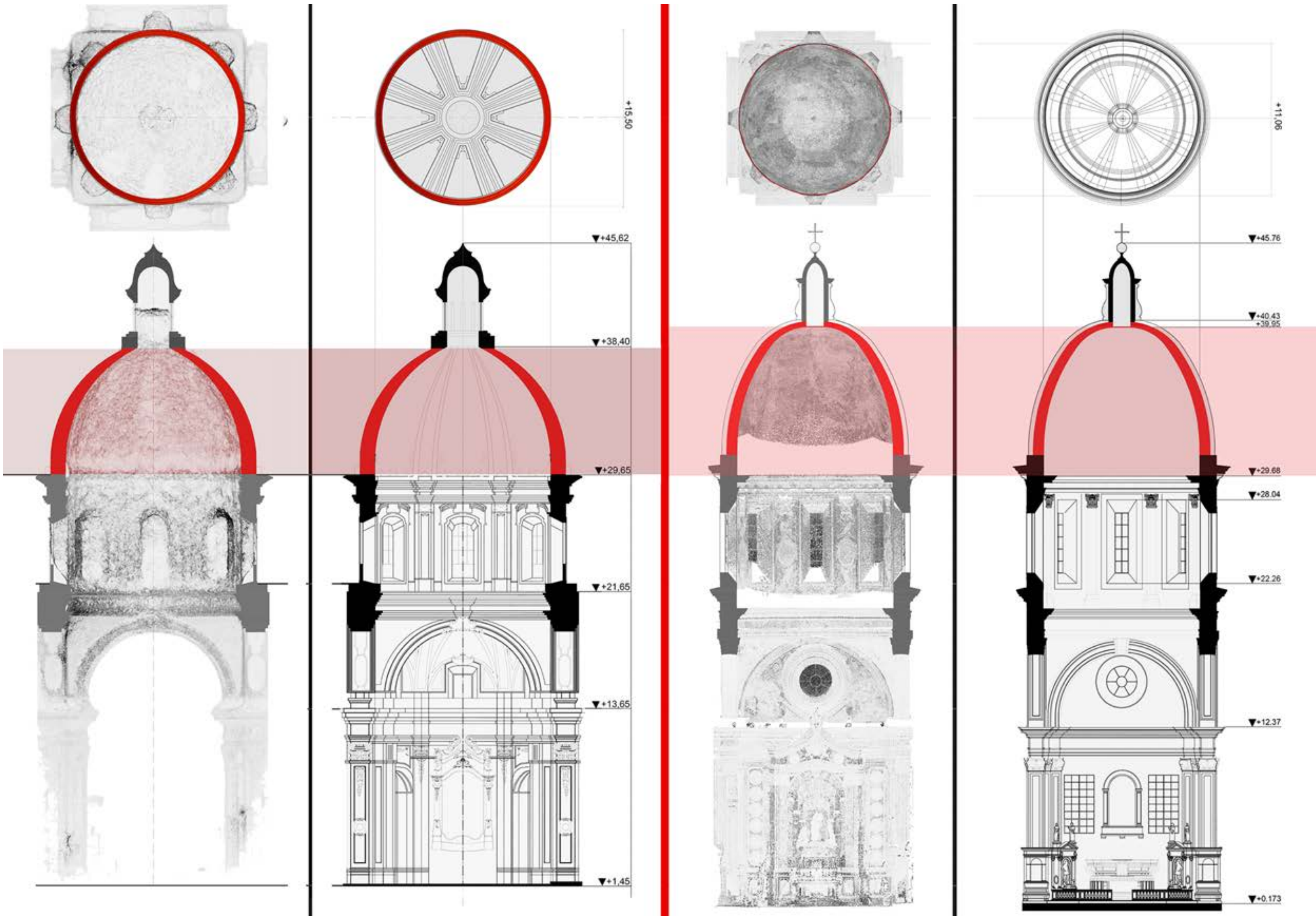

Fig. 2 - Semantic geometric segmentation: vault intrados analysis. Comparison between the domes of S. Maria di Costantinopoli Church (pointed arch) and of S. Caterina a Formiello Church in Naples (polycentric curve).

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automatic and/or semi-automatic parametric processes, based on digitalization and classification of historical architectural heritage elements, represent a possible solution to simplify analysis and modelling activities. According to these premises, the main goal of our testing activity is not to automate the interpretation of a shape, but it aims at simplifying graphics and restitution. In this process the input data are selected by the operator (*human driven* approach) while graphing becomes automatic (semi-automatic process). The result is a 'semi-ideal' model, that is a simplified object close to the real element. Moreover, simplification is useful for multiple purposes, especially to model complex decorative apparatuses that need a defined and continuous support for their distribution.
According to these premises, we are testing specific VPL tool about shape optimization (Khan, 2015) to study complex historic/existent architectural shapes merging critical human-driven interpretation, based on geometric rules knowledge and GAs. Current VPL tools (*e.g. Galapagos-GH* component, *Octopus-Grasshopper plug-in, Rhino, McNeel*) allow to test *Evolutionary Theories* (Darwin, 1859) for *problem solving* and *decision making*. According to a *human driven* approach, the operator defines *GENOME, FUNCTION* and *FITNESS* to drive the *Evolutionary Solver* (*ready to use*) towards optimized solutions. We have tested these tools to interpret and restitute curves from reality-based profiles used in domes surface generation. More specifically, our advance consists in testing GAs tools able to automatic identification open (polycentric or pointed) or closed (oval or elliptic) curves, starting from planar sections (reality based irregular polylines) extracted from point clouds (Fig. 1).
These applications can be useful to parameterize complex architectural systems (CAD/VPL/HBIM interoperability) and to support AI processes applied to point clouds. Moreover, geometric interpretation supports simplification of existent architectural shapes even according to current BIM/HBIM LOIN (*Level of Information Need*) oriented approaches.

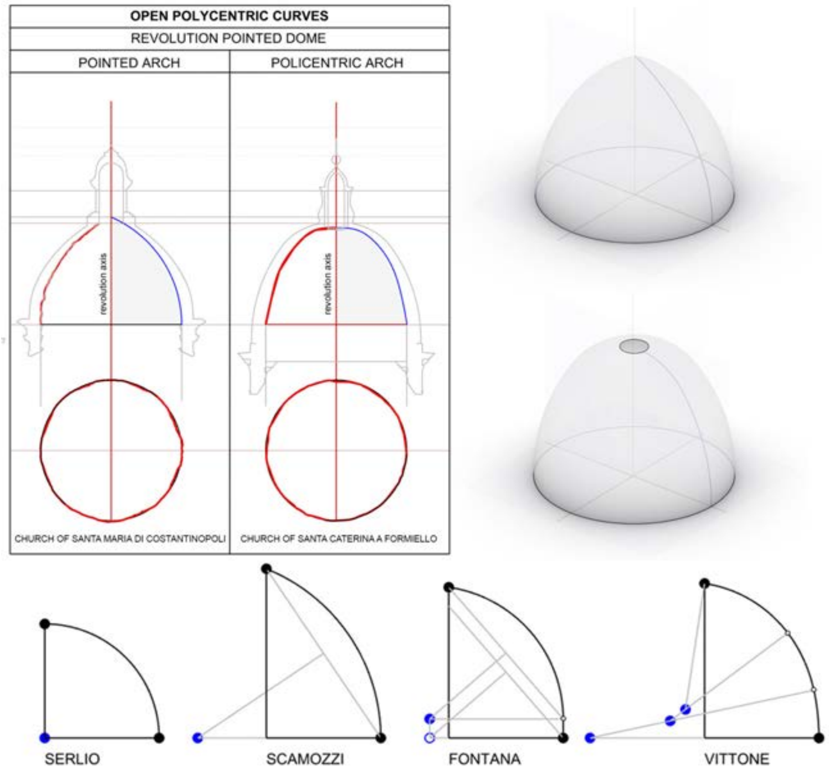

Fig. 3 - Shape analysis and geometric-semantic classification of pointed domes: 'pointed arches' and polycentric profiles.

2. GEOMETRY AS HEURISTIC TOOLS. HISTORIC SHAPES CLASSIFICATION

The analysis of an architectural systems is not always a simple procedure. Objective data are often combined with subjective interpretations about related drawings (Canciani, 2015). Therefore, it is interesting to test different research approaches aimed at recognizing forms based on theoretical and practical implications involved in geometric interpretation and construction, and on the heuristic role of digital modelling (Galizia et al., 2012).

The main goal is to simplify historical architecture digitalization process, whose complexity is due to the lack of design and ancient constructive information. "*The geometric model, by defining the surfaces delimiting shapes and spaces, allows to represent and manipulate Architecture. It contains both figurative and material components*" (Docci 2011, p.15). According to these premises, in our previous research activity, we have defined an experimental *Scan to HBIM workflow* aimed at generating a dataset of revolution domes based on geometric rules from historical treatises (Ca-

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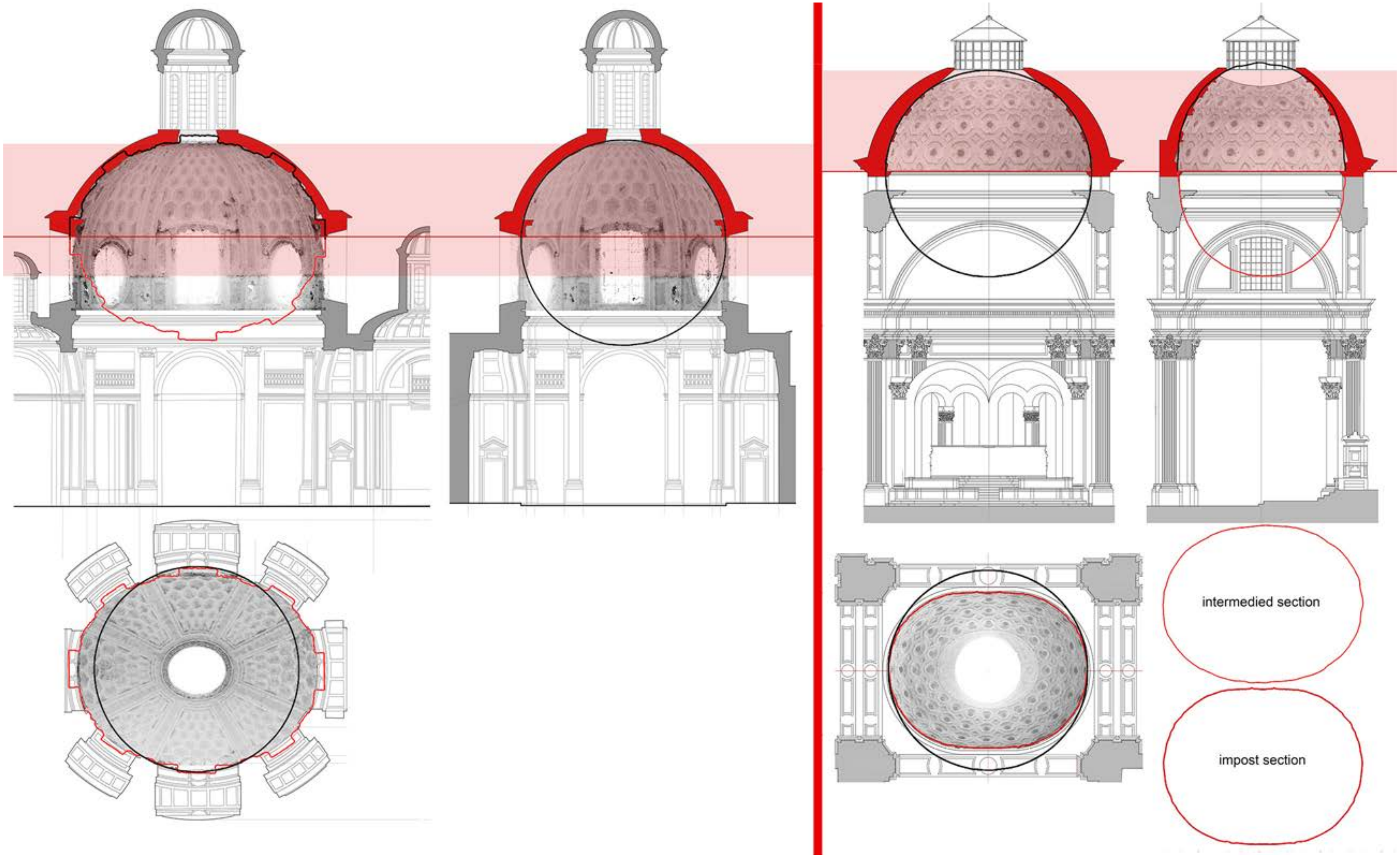

Fig. 4 - Semantic-geometric segmentation: vault intrados analysis. Comparison between the domes of Padri della Missione Church (ellipsoidal dome) and of S. Giovanni Maggiore Church (oval dome) in Naples.

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pone et al., 2019a), and we have defined tools for decorative apparatuses procedural modelling (Capone et al., 2019b).
Therefore, to correctly model these architectural and structural elements, first we need to individuate and extract the main profiles geometric genesis, according to semantic decomposition and shape classification. Research topic is the analysis of domes geometric nature that moves from theoretical knowledge and analysis of case studies.
As we know, domes are double curvature surfaces: the most common domes spring from a circular base and for that we call them circular domes, even if their cross-section is not circular. Circular convex domes are revolution surfaces generated by rotating a meridian curve around a vertical axis. We have analyzed geometric genesis of the meridian curve and we have identified three kinds of domes: domes with meridian curve composed by a quarter of circle, in this case they are hemispherical dome, domes with meridian curve composed by a 'pointed arch' and domes meridian curve composed by a polycentric arch (figg. 2, 3). Other kinds of domes are the oval domes, that can be defined as domes whose plan or vertical sections have an oval form (polycentric curve) and ellipsoidal domes, that can be defined as domes whose plan or vertical sections have an elliptic form. Revolution oval domes can be created by rotating a plan curve (the generating curve) around a horizontal axis or by interpolating of curves networks (horizontal sections and radial sections) (figg. 4, 5). In this contribution, we show results about GAs testing activity to semi-automatically curves identification and restitution about open polycentric curves, closed polycentric curves and elliptical sections.

Polycentric curves are curves made of a finite number of arcs of circle subsequently connected with a common tangent in the common point, they can be closed (ovals) or open curves. Their use in architecture has been widespread: usually the oval plan has been used for churches, while closed polycentric curve has been used as meridian curve to generate domes of revolution. There are some different ways to construct ovals or open polycentric arches based on different geometric rules. The oval is not determined if we only know the axes and a pointed or polycentric arch is not uniquely determined if we only know span and rise. According to these constructions, an oval is not determined if we only know the axes and a pointed or polycentric arch is not uniquely determined if we only know span and rise. Therefore, by changing position and division of these elements, it is possible to generate infinite configurations.

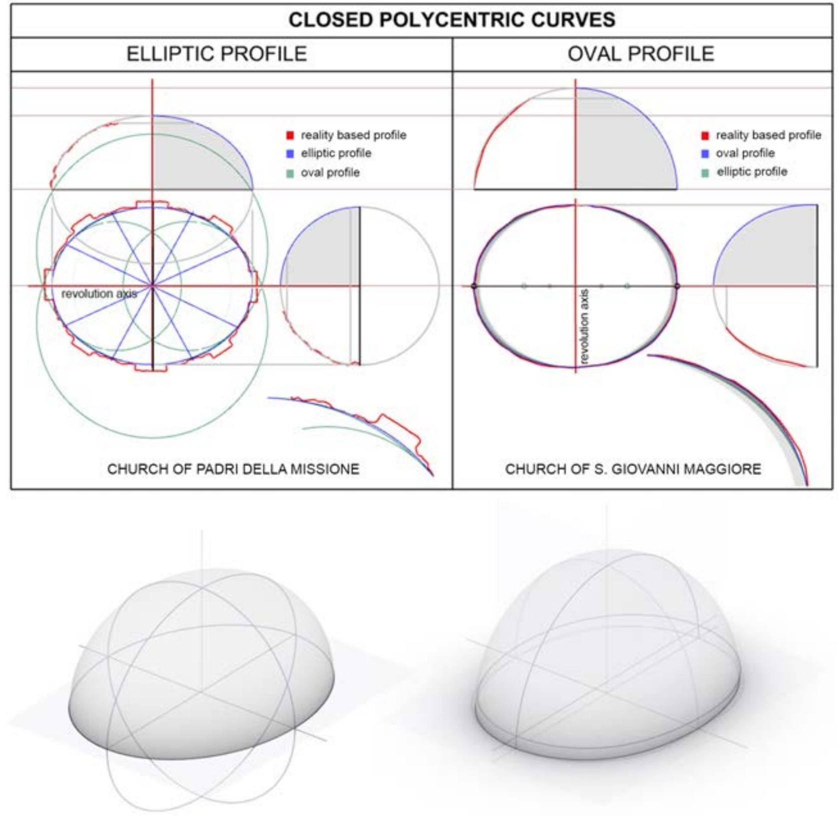

Fig. 5 - Shape analysis: comparison between ellipsoidal and oval revolution domes.

Although ellipse and oval are analytically different curves, they are similar and, for this reason, they are often confused (Migliari, 1995).
The ellipse is a conical curve already defined in classical age by Euclide as the planar section of a right cone with a plane perpendicular to its side [1]. Instead, Apollonius defines the cone as an infinite and not necessarily straight surface, composed of two opposite and symmetrical portions with vertex in common. Furthermore, about its definitions, different inclinations of the section plane allow to identify three different conics [2].

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In mathematics, an ellipse is a plane curve surrounding two focal points, such that for all points on the curve, the sum of the two distances to the focal points is a constant.

The oval is a planar and closed polycentric curve without univocal definition and delimiting a convex region. In the seventeenth and eighteenth centuries (Canciani, 2015), treatises by A. Bosse (1643) and by A. F. Frézier (1737) contributed to diffuse oval in architectural design. More particularly, the *First Book of Architecture*, by Sebastiano Serlio (1545), is one of the most impor-

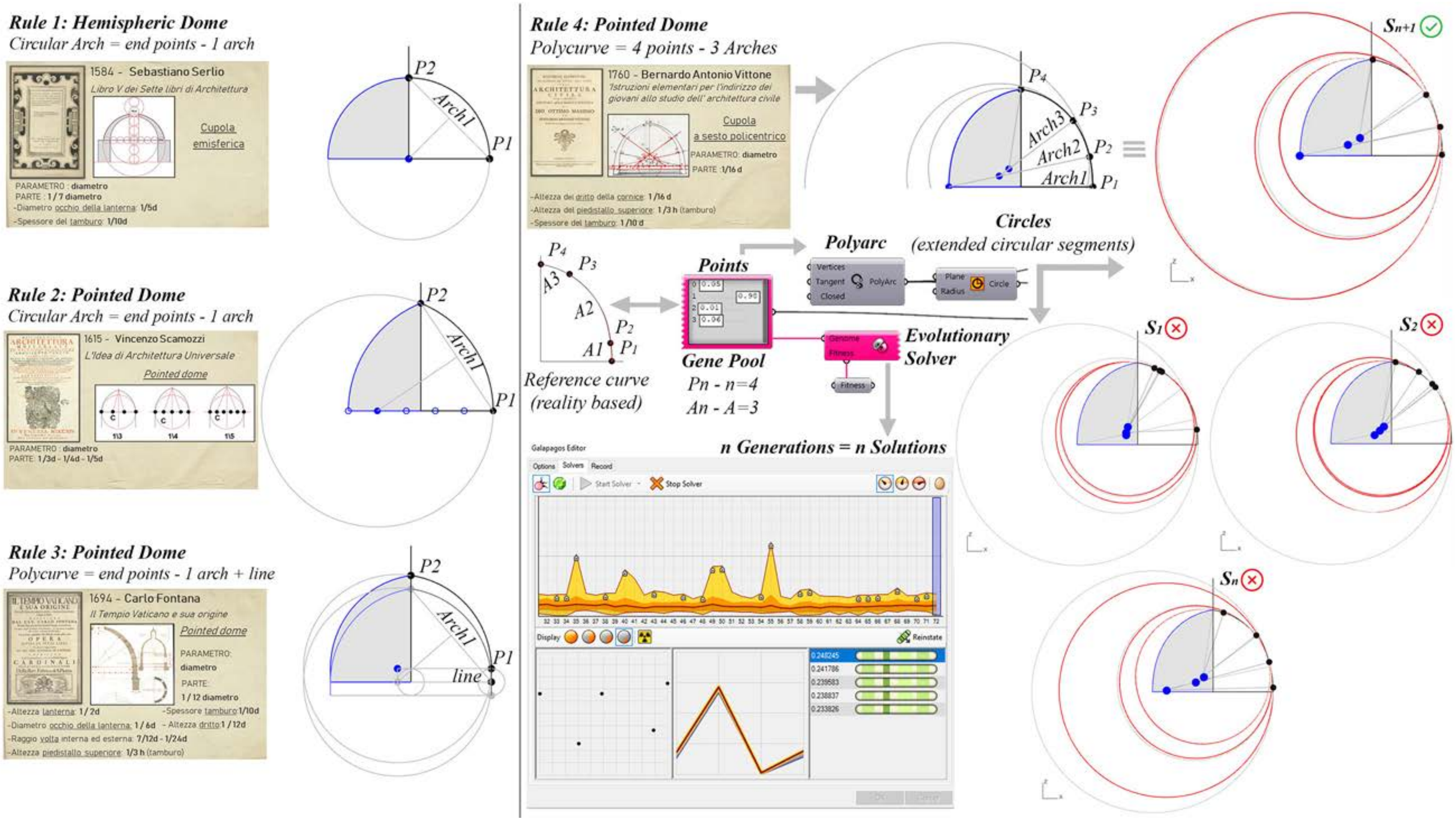

Fig. 6 - On the left: VPL translation of ancient geometric rules (from center to points along curve); on the right: GA to search the rule that best fits a reality-based curve (from point along curves to center layout).

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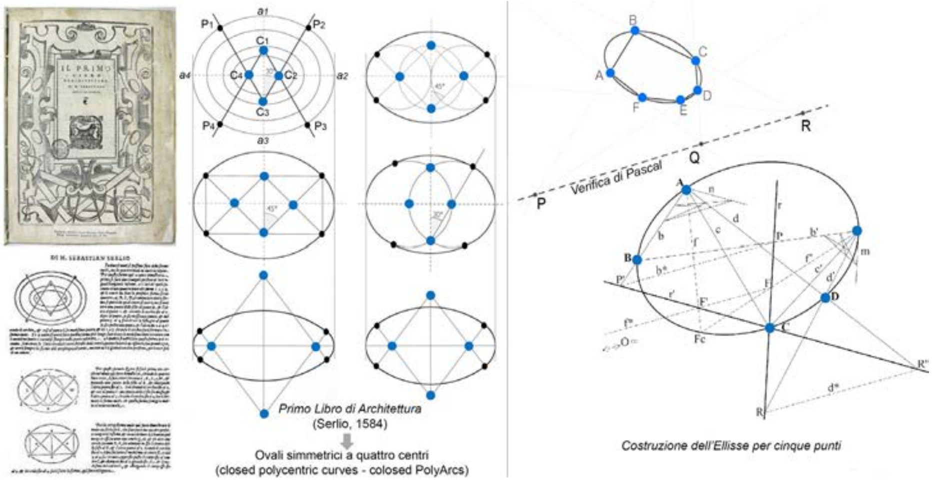

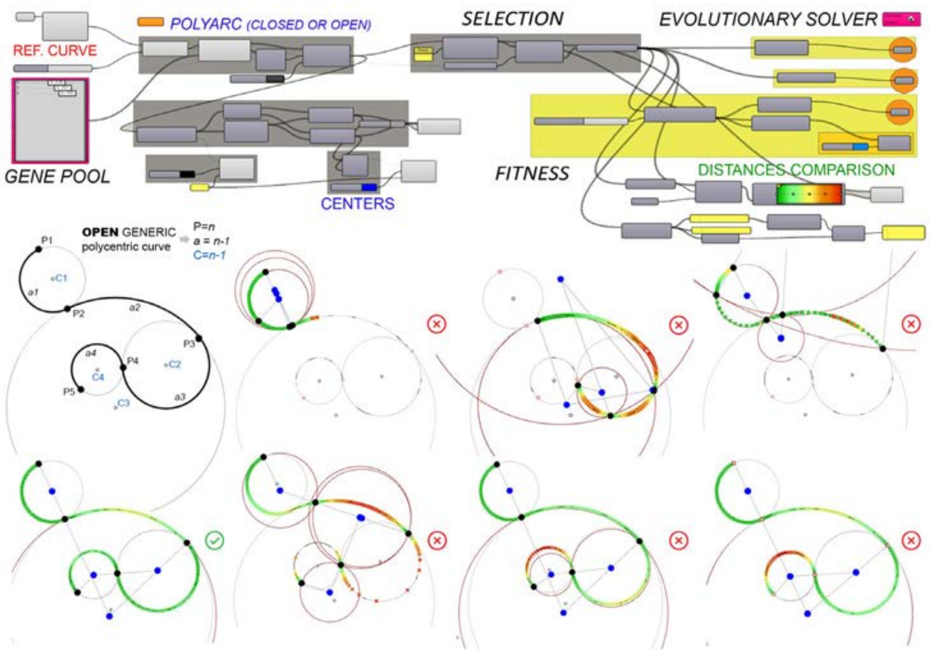

tant references for studying ovals (Santagati et al., 2018; Zerlenga et al., 2018; Canciani, 2015; Magnano di San Lio et al., 2012, Dotto, 2002) unlike the ellipse, starting from the same axes, it is possible to generate infinite ovals and a single ellipse (Dotto 2002, p.14).

Starting from rules, our aim is to test a process that allows to generate the polycentric curve (closed or open) nearest to reality-based curves. In our research we analyzed some of the geometric methods described in treatises with the aim to compare the theoretical model (ideal model) with the realty-based model from point cloud. The comparison allows us to do a critical 3D data interpretation based on geometric rules (figg. 6, 7). In the first step of our research, we have developed a traditional parametric model based on geometric rules, then we have defined a process based on GAs driven interpretation. The *Genetic Solver* identifies the optimal distribution of points (*COMBINATION*) by minimizing or maximizing different comparison conditions between the two curves (*FITNESS*), in order to optimize their overlapping (minimizing the sum of the distances between the ideal curve and the reference one, minimizing the average distance between curves or maximizing the identification of the number of ideal points whose distance from the reference curve is less than or equal to a given limit value), selecting the best solutions (*SELECTION*) towards the optimal solution.

3. METHODOLOGY AND TOOLS. FROM GEOMETRIC RULES TO GENERATIVE ALGORITHMS

The main advance of our research consists in testing transition from traditional algorithms to GAs used to simplify some curves analysis and

Fig . 7 - On the left: VPL translation of ancient geometric rules to define oval shapes (Serlio, 1584); on the right: ellipse verification (Pascal's rule) and construction (five points).

Fig. 8 - GA testing to interpret and define open polycentric profiles. Points colors explicit the comparison between ideal curve and 'semi-ideal' according to their distances.

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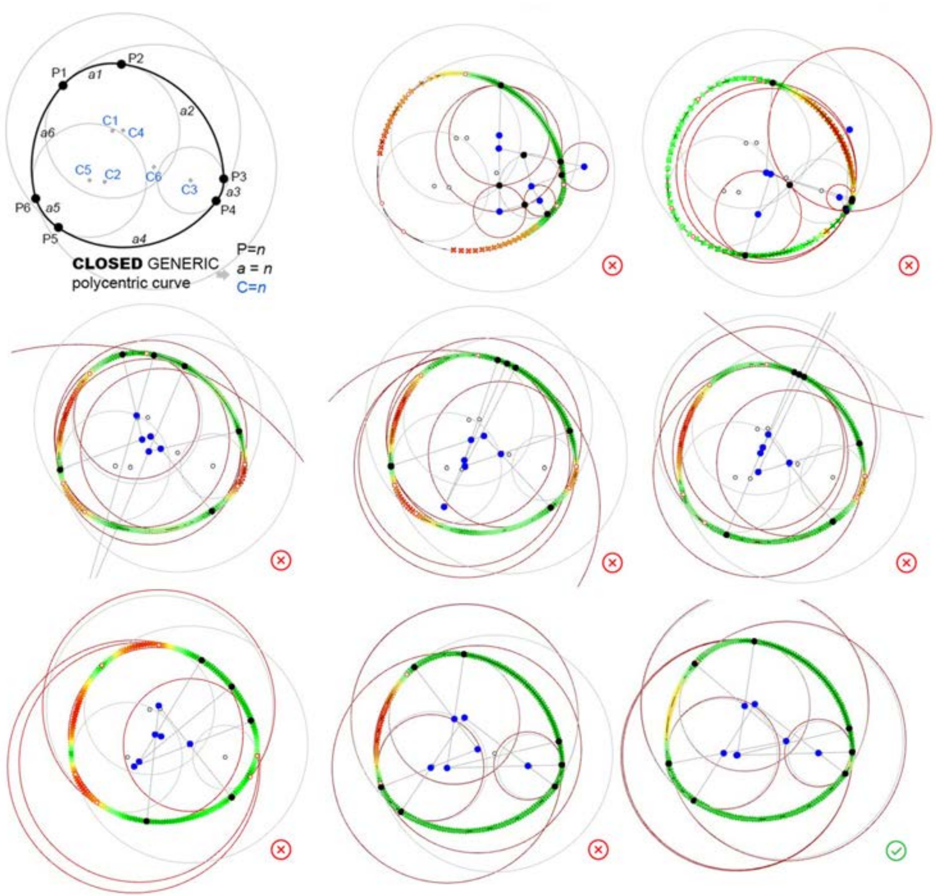

Fig. 9 - GA definition and testing to interpret and define closed polycentric profiles. Points colors explicit the comparison between ideal curve and 'semi-ideal' according to their distances.

especially polycentric curves, ovals and ellipses restitution. The process is based on AI principles to determine the choices to reach a certain goal among different possibilities (*e.g.*, it can be used to determine the curve that best fit the real curve).

Our research is based on domes study and testing activities, the main steps are:

1. geometric rules definition from treatises;
2. definition of traditional algorithmic-generative modelling (VPL) to translate rules from Treatises and overlap/comparison between reality-based elements (point cloud) and ideal models (Capone et al., 2019);
3. comparison between reality-based curve and algorithmic model according to critical *human-driven* interpretation;
4. GA driven interpretation to automatically find "the best solution", *i.e.* the ideal or 'semi-ideal' curve nearest to the reference curve from point cloud;
5. comparison between different approaches and tools (VPL/C++) to identify ideal curves and surfaces that best describe and fit point clouds segments (Lanzara et al., 2019).

GAs are specific tools simulating natural selection process to solve both constrained and unconstrained optimization problems. GAs allow to solve a variety of optimization problems not well suited for standard optimization algorithms, including problems in which the objective function is discontinuous, non-differentiable, stochastic or highly nonlinear. Current VPL generative-algorithmic tools (*e.g. Galapagos*) allow to test *Evolutionary Theory* (Darwin, 1859) to support *problem solving* and *decision-making* processes. According to a *human driven* approach, the identification of parameters to define *GENOME, FUNCTION* and *FITNESS* allows to drive the *Evolutionary Solver* towards optimized solutions. The GAs main steps are:

- identification of starting population of a certain number of individuals or 'genome' (*GENE POOL*);
- setting a *FITNESS* to search for optimized results according to specific criteria;
- possible introduction of additional operators (as occurs in nature), including *MUTATIONS* or *CROSSOVER*;
- possible setting of a *TERMINATION TIME CRITERIA*.

Our definition allows generic polycentric profiles and analytic curves restitution: it defines geometric functions starting from a random sequence of points along the reference curve. More specifically, the comparison between real curves

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and ideal rules allows to train GAs starting from a random number of points (*GENE POOL, i.e. variables*) along the irregular reference polyline to build the analytic curve that best fits the reference one (*SELECTION*), according to a specific condition (*FITNESS*).
About curves interpretation using Gas, we can consider these *ready to use* components:

1. *PolyArc* (*GH component*) is aimed at drawing a sequence of circular segments starting from a series of points (*vertices*) by imposing tangents continuity (*tangents*) along the reality-based profile and it allows to restitute open or closed polycentric profiles; GA calculates the optimal position of the *PolyArc endpoints* along the real profile to extrapolate a 'semi-ideal' polycentric curve;
2. *InEllipse* (*GH component*) is aimed at drawing an ellipse inscribed into a triangle: the triangle edges are tangent to the inscribed ellipse (*Steiner's InEllipse* rule), therefore to the reference curve (considered as a hypothetical ellipse) in three points. Likewise *PolyArc*, also this component requires only points along reference curve as input elements. For closed curves, ovals and ellipse, we selected analytic rules to restitute these curves only from points, by comparing conditions required for traditional algorithms or 'manual' constructions with those required to instruct GAs. This step is important to refer functions only to the number of points along the curves. We can clarify the difference between parametric traditional model and GA from geometric condition comparison.

Traditional graphic/algorithmic geometric conditions:

- radius and angular amplitude for a circular arch (then circle);
- a minimum of two points and a tangent for circular arch (then circle);
- a minimum of three points not aligned for circle;
- a minimum of four non-aligned centers for oval (closed or open);
- a minimum of two non-aligned centers for *PolyArc* (*GH component*), from circle to generic polycentric;
- the center and two axes for ellipse;
- a minimum of five points not aligned for ellipse.

Geometric conditions for GA (*GENE POOL* composed by parametric points settable through a numeric slider):

- a minimum of two non-aligned centers for *PolyArc* (*GH component*), from circle to generic polycentric;
- a minimum of five points not aligned for ellipse;
- a minimum of three points for ellipse (*InEllipse, GH component, Rhino* for *Steiner InEllipse*)

The result is a 'semi-ideal' profile, near to an ideal solution (rule), close to the reality-based curve: it is directly built on points belonging to the reference curve, but it also represents an optimized solution because it is reconstructed as an analytic curve. Thus, the result is not known or imposed *a priori*. The ideal solution, especially in this field, rarely exist: a real architectural element is not "perfect" as a technical drawing, due to the transformations and building defects.

3. GAs FOR OPEN AND CLOSED POLYCENTRIC CURVES INTERPRETATION

About polycentric curves interpretation and restitution, our advance consists in testing GAs able to automatic identification open (polycentric or pointed) curves (fig. 8) or closed (generic or oval) curves (fig. 9), starting from planar sections (reality based irregular polylines) extracted from point clouds.
GAs has been here applied for interpretation and restitution of the vertical sections of pointed and polycentric circular domes (open *PolyArcs*), and of the impost curves and the horizontal sections used for oval domes constructions (closed and symmetric *PolyArcs*).

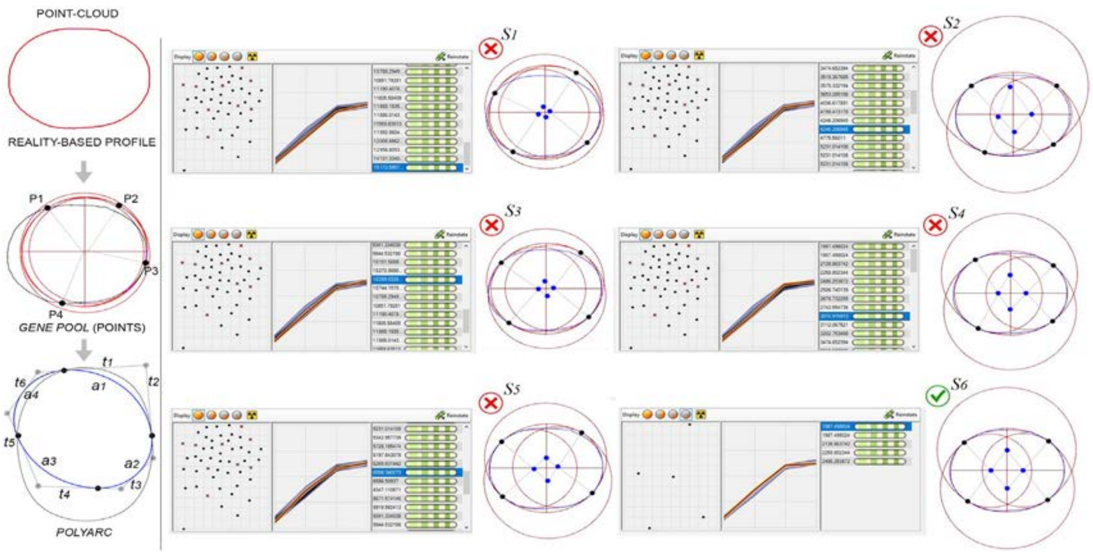

Fig. 10 - GA iteratively evolves towards the optimal ones. Each iteration corresponds to a generation that progressively best fits the reference profile.

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Circular domes are revolution surfaces generated by rotating a meridian curve around a vertical axis, the meridian curve can be composed by a quarter of circle, in this case they are 'hemispherical domes', by a pointed arc or by a polycentric arc, 'pointed domes'.

There are few cases of domes generated by parabolic, elliptic, or catenary curves. First step is to build a parametric model based on treaties rules. The main effort is to construct a unique parametric model to relate the different geometric procedures. We have done the parametric model using the different meridian curves based on geometric curves by Serlio and by Palladio (circular section), by Fontana and by Vittone (open polycentric curve) and by Scamozzi ('pointed arch'). Therefore, we have defined a unique algorithmic-generative model to con-

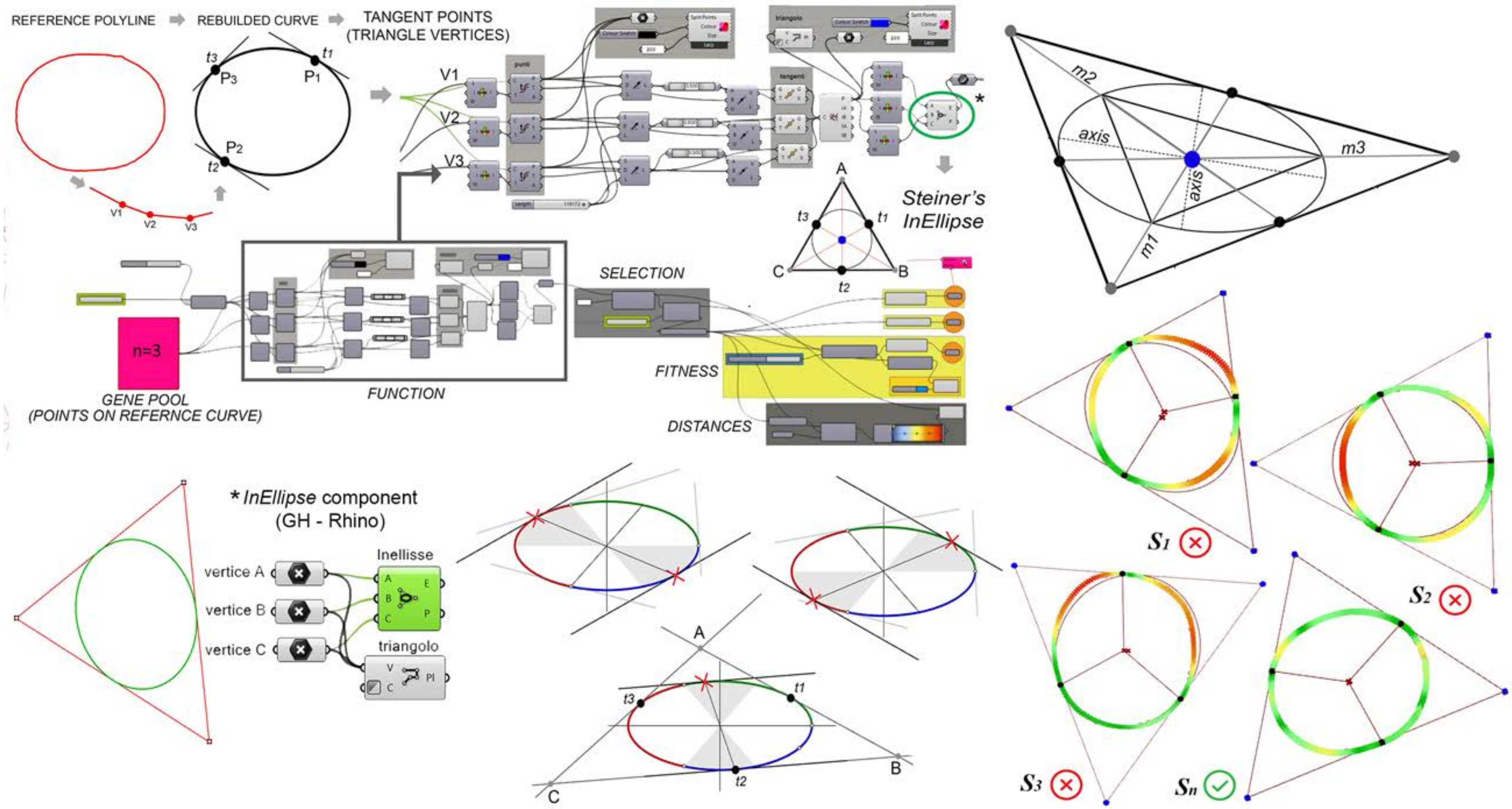

Fig. 11 - GA definition and testing to interpret and define elliptic profiles according to *Steiner's InEllipse* rule.

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Fig. 12 - The dome of the Church of S. Maria di Costantinopoli and the dome of the Church of S. Caterina a Formiello (Naples).

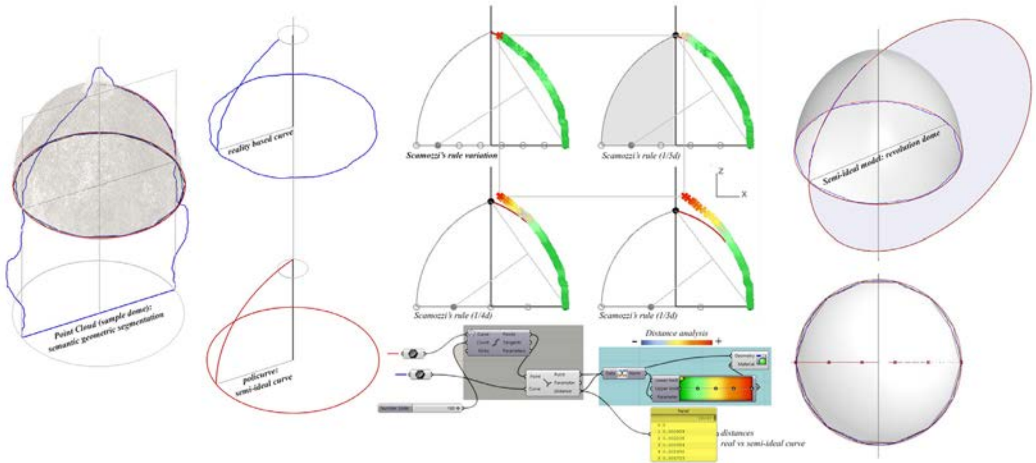

Fig. 13 - GA application: comparison between reality based, 'semi-ideal' and ideal hypothesis (pointed arch - Scamozzi's rule variation) to interpret the profile of S. Maria di Costantinopoli dome.

nect different processes according to the same parameter: the dome diameter (centers of circular arches are along diameter).
Oval domes can be generated in two different way: a revolution surface, generated by rotation of horizontal planar section that is half of an oval, or surface generated interpolating a curves network (horizontal and meridian sections). We can define an oval as a closed symmetric *PolyArc* and we can draw it dividing the axis according to different rules (Serlio 1584). By inverting this rules logic (from centers to points along curve) it is possible to trace the layout of the centers starting from the input points from the *PolyArc* that best fits the reality-based profile (from points along curve to the centers). *PolyArc* is composed by an open or closed sequence of tangential circular arches (*Boolean Toggle: True/False*). Therefore, the main goal is to automate the distribution of n points along the reality-based curve so that the 'semi-ideal' curve is closest to the reference profile and the layout of the centers coincides

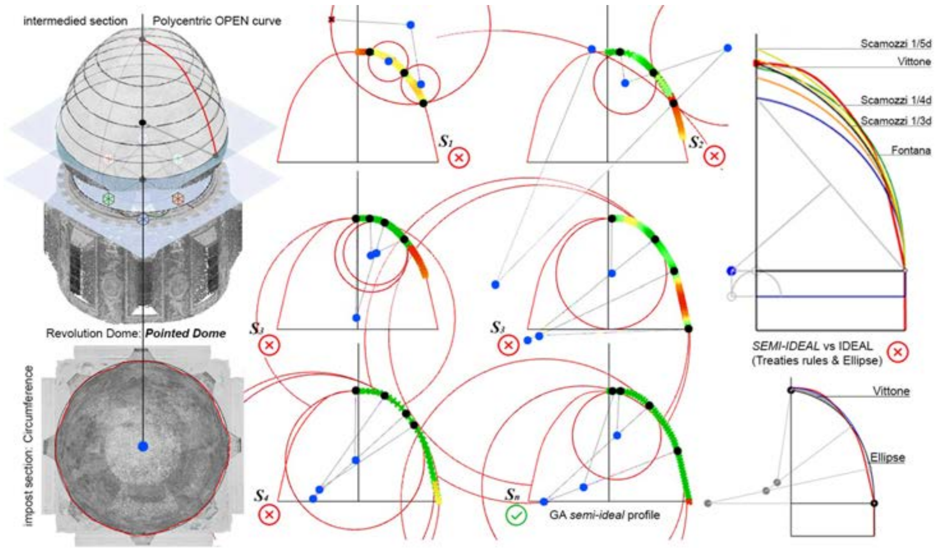

Fig. 14 - GA application: comparison between reality based, 'semi-ideal' and ideal hypothesis (polycentric profile – Vittone's rule variation) to interpret the profile of of S. Caterina a Formiello dome.

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with the rule that best interprets it. The number of points can be random or deriving from a critical interpretation: symmetrical or radial distributions of architectural elements simplifies the decomposition of the profile into its segments and allows hypotheses about the kind of oval that best describes it. More specifically, the *GENE POOL* is composed of numeric sliders (interval of definition [-1, +1]). Points along re-parameterized reference profile are assumed as numeric parameters. If the profile is closed, the number of points (*GENE POOL + Seam*) is equal to the number of arcs; if it is open, the number of points is equal to the number of arches/centers +1. Numeric sliders allow random selection of parametric values along reference curve: increasing decimals, also the number of infinitesimal intervals, then of points positions, increase and the result will be more accurate. The *Evolutionary Solver* identifies the optimal combination of points by minimizing the distance between points (mass addition) along reality-based curve and *PolyArc* (*FITNESS*), to optimize their overlap.

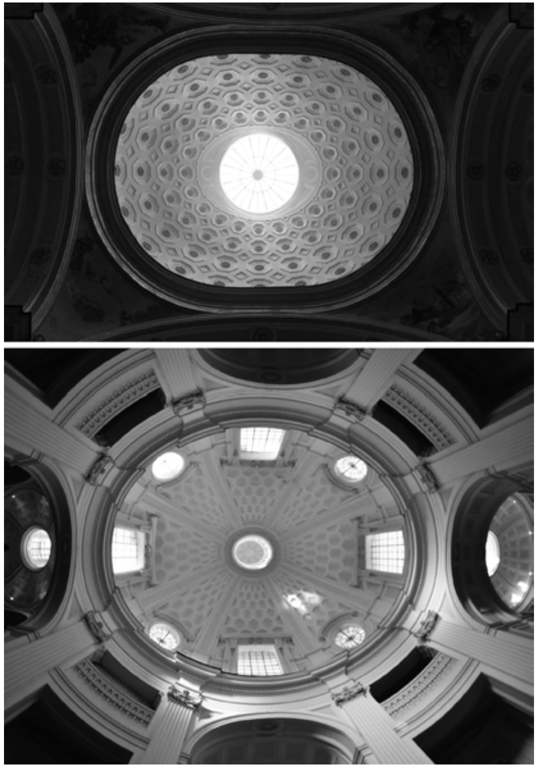

Fig. 15 - The dome of the Church of S. Giovanni Maggiore and the dome of the Church of Padri della Missione (Naples).

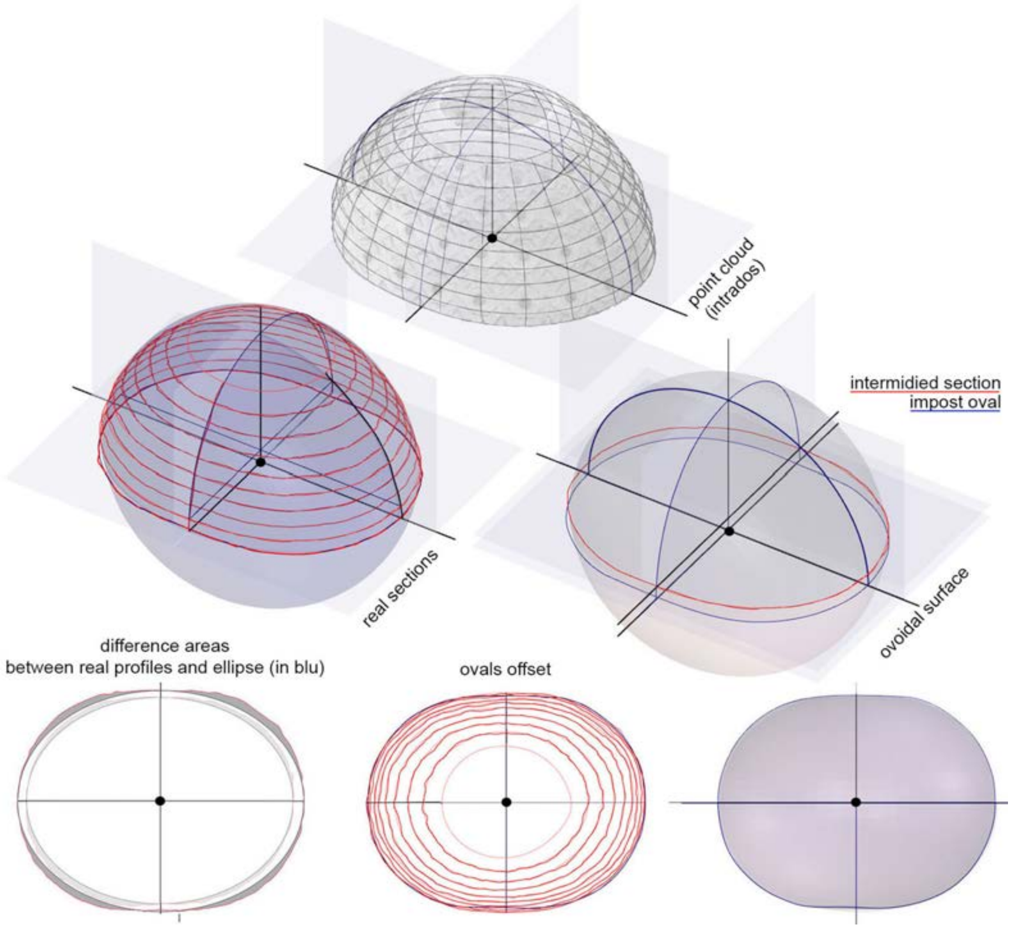

Fig. 16 - Contour of reality based textured mesh of the dome intrados of San Giovanni Maggiore church (horizontal and radial vertical sections) and modelling of revolution oval shape. Comparison between reality based and 'semi-ideal' models.

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FITNESS defined and tested to select the optimal solutions allow to minimize the sum of the distances between the ideal curve and the reference one, to minimize the distance between curves and to maximize the identification of points whose distance from the reference curve is smaller or equal to a given limit value. The *Solver* creates a first population characterized by a random generation, minimizing or maximizing established *FITNESS* and iteratively evolves the solutions towards the optimal ones. Each iteration corresponds to a generation, or possible solution, expressed as *PolyArc* that progressively best fits the reference curve. Given three points along each irregular reference segment, it is possible to extract infinite circles: GA calculates their optimal position by minimizing the distance between 'semi-ideal' *PolyArc* and reference profile to correctly interpret it (fig. 10).

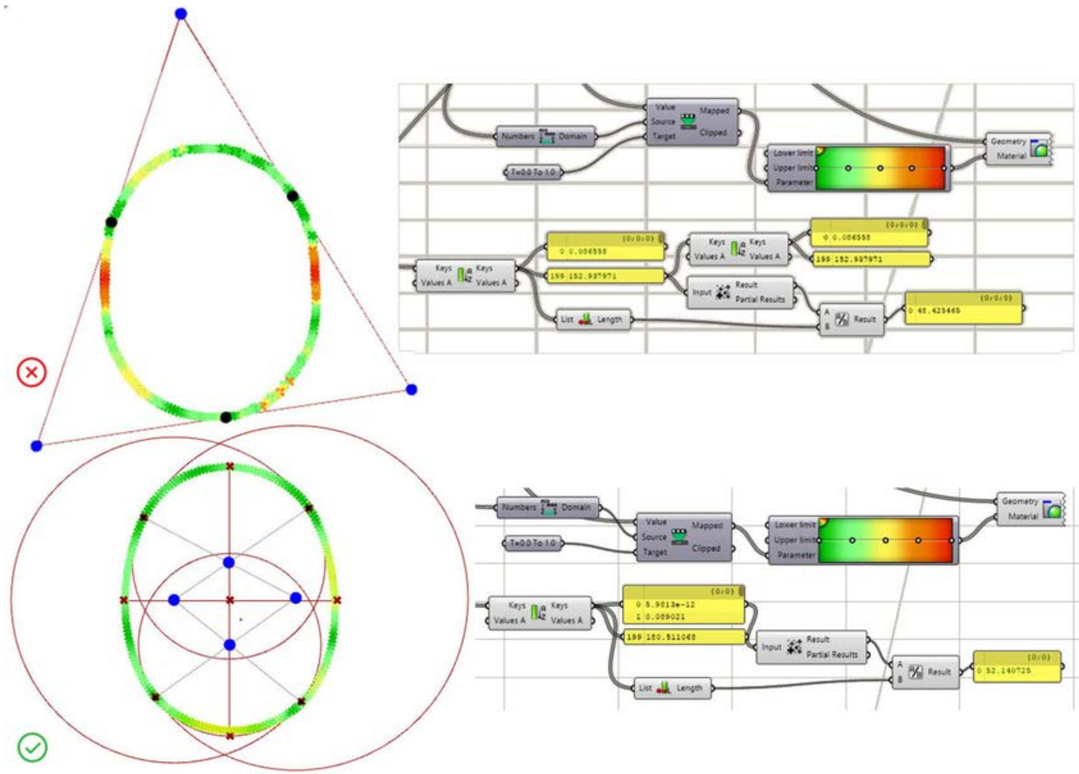

Fig. 17 - Comparison between the reality based intermediate sections of the San Giovanni Maggiore dome, elliptical and oval profiles: oval shape best fits the reference curves.

4. GAs FOR ELLIPSE INTERPRETATION

Domes are revolution elliptical domes if the planimetric or vertical profiles are ellipses. According to geometric construction of curves from points, we know that we can draw an ellipse starting from five unaligned points along the curve (Di Paola, 2011). In particular, in this case, we instructed GA to search for the ellipse that best fits a reality-based curve using *InEllipse* component (*GH, Rhino*). This component returns *Steiner's Ellipses* starting from three coplanar and not aligned points coinciding with vertices of the circumscribed triangle. Steiner's *InEllipse* is the *inconic* (tangent a triangle at the midpoints of its three sides). More generally, an *inconic* is a conic section that is tangent to all sides of a triangle.
The *InEllipse* component (*GH, Rhino*) requires as input the three vertices of the triangle circumscribed to the curve. According to Steiner's rule, the sides of the triangle are tangent to the curve: for each triangle there is one and only one *Steiner ellipse* (Kalman, 2008). Furthermore, it corresponds to the circumellipse of the medial triangle and its center coincides with the barycenter of the triangle. Therefore, it is necessary to explicit the component by extracting three tangents in three points (*GENE POOL*) along the irregular reality-based curve (polyline), rebuilt as interpolate curve. By intersecting them it is possible to identify the three vertices of the triangle required as the only inputs from the *InEllipse* algorithmic component. The *Evolutionary Solver* provides the *inconic* closest to the reference-curve, minimizing their distances (*FITNESS*).
The GENE POOL is composed of three points and the three numeric bars are defined into three different intervals: [0, +0.33], [+0.34, +0.66] and [+0.67, +1]. The reparametrized curve is defined in the interval [0, 1]. Along the ellipse, the pairs of tangents extracted from the *endpoints* of the infinite diameters are parallel, therefore they do not incident in the proper points required to identify the vertices

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of the circumscribed triangle. This condition can be limited by dividing the interval of definition into three parts: each point must belong to a different range. However, this condition is *work in progress* to improve the definition (fig. 11). Moreover, according to an ideal condition (impossible condition, reality-based curves are never perfect ellipses), points from the 'semi-ideal' curve and along *InEllipse* (triangle tangency) are coincident.

5. COMPARISONS

One of the main advantages of this definition is that the only parameter we need is the number of points. The same definition allows to draw closed and open 'semi-ideal' polycentric curves, we can simply communicate this condition by using a *Boolean Toggle* (*True/False*). We can analyze, define and restitute all kind of polycentric profile using only one definition.

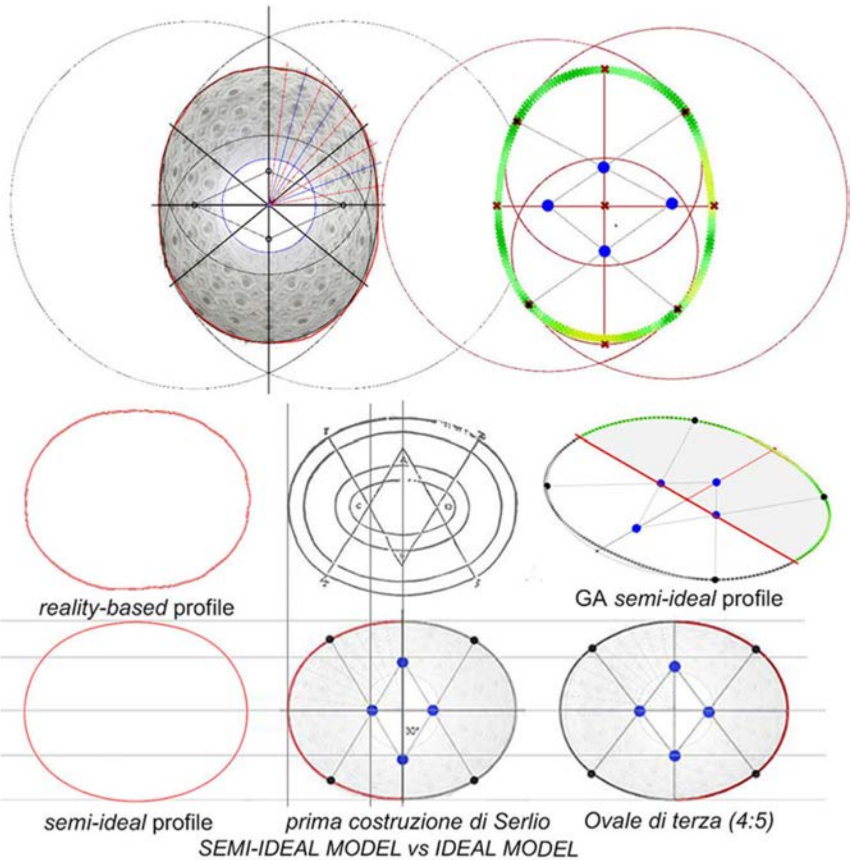

Fig. 18 - Comparison between the 'semi-ideal' oval intermediate sections of the San Giovanni Maggiore dome and ideal ovals from treaty rules (Serlio, 1584).

Another important advantage is to use the same *GENE POOL* (points along *reality-based curve*) to interpret and restitute different curves, from generic open or closed polycentric curves to ellipse. A chromatic gradient allows to recognize the points along reality-based profile according to their distances from provided 'semi-ideal' curve (*PolyArc* or *InEllipse*) and the corresponding *ideal curve*: for a value of 0, the points are green; for higher values, the points are red.
It is difficult to have a regular, symmetric reference profile: for this reason, optimal *PolyArcs* are defined 'semi-ideal', while the ellipse is unique. GAs do not find a single best solution, but a range of optimal solutions towards the ideal condition, the *ideal curve*.

6. GAs APPLICATIONS

The theoretical ideal models, defined according to treaties study (traditional algorithms), have been used to compare different approaches: we have tested GA on some emblematic case studies to verify the correspondence between previous and new solutions (Capone et al. 2019.a). This comparison allows to test and validate the process according to its reliability. All the intrados survey are defined using Terrestrial Digital Photogrammetry - SfM.
GAs applications to analyze the shapes of the curves involved in the geometric genesis of the Domes of the Basilica of San Giovanni Maggiore and of Church of Padri della Missione in Naples that are particularly interesting due to the small difference between oval and elliptic interpretation of their profiles.

7. SAMPLE 1. GAs FOR OPEN POLYARCS AS POLYCENTRIC OR POINTED ARCHES

The GA to interpret and draw pointed arches was tested on the revolution *pointed dome* of the Church of Santa Maria di Costantinopoli in Naples, while GA to interpret and draw policentric curves was tested on the revolution *pointed dome* of the Church of Santa Caterina a Formiello in Naples (fig. 12).

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By comparing Serlio, Fontana, Vittone and Scamozzi's parametric rules and the reality-based sections, we can state that the dome of S. Maria of Costantinopoli is generated by a pointed arch, but the *ratios* provided by Scamozzi for the position of the center along the diameter (1/3d, 1/4d and 1/5d) were not used the best position that we have defining overlapping the two curves is equal to 1/7d (fig. 13). Even the intrados profile of the dome of the Church of Santa Caterina a Formiello does not exactly correspond to any cited rules (fig. 14).

8. SAMPLE 2. GAs FOR CLOSED CURVES: OVAL VS ELLIPSE

The GA for oval interpretation has been tested to verify the impost and intermediate sections involved in the oval coffered dome of the Basilica of San Giovanni Maggiore in Naples, while for ellipse interpretation we have verified the profiles involved in the ellipsoidal dome of the Church of Padri della Missione. Both these churches are in Naples (fig. 15). Among the most important basilicas of Naples, this church is characterized by a latin cross plan with three naves, a transept, a dome covering the presbytery area and a semi-dome covering the apse, both coffered. Dating to the year 324 AD (foundation age), the most important and recent transformations was introduced by Dionisio Lazzari, who redesigned the dome during the seventeenth century.
A double symmetrical oval (four centers) is clearly recognizable with the naked eye by observing the extrusion profile of the molded frame crowning the drum and corresponding to the impost curve of the coffered intrados of the dome (fig. 16). However, the oval profiles of the intermediate sections are very similar to ellipses: therefore, correspondence with ellipse was also verified (fig. 17). Unlike the impost profile, the algorithm calculates a minimal difference between intermediate sections and ellipse. However, the presence of lacunars would confirm the oval shape for their sections: in fact, oval allows regular offset. Figure 18 shows comparisons between 'semi-ideal' ovals and Serlio's rules (1584).
The GA for ellipse recognition was tested to verify the shape of the impost and vertical profiles involved in the geometric genesis (intrados) of dome of the Church of Padri della Missione in Naples (fig. 19). The church, a reference model for the religious architecture of the city, is one of the masterpieces of Luigi Vanvitelli, who worked on it during the eighteenth century. Over the centuries, scholars confirm the elliptical shape of its central plan and coffered dome (Capone 1999, p.137).
As for the dome of the Basilica of San Giovanni Maggiore, the algorithm calculates a minimal difference between ellipse and oval. Moreover, the presence of ribs could confirm intrados subdivision, then a possible oval profile. However, a direct comparison between curves demonstrates the best correspondence of the reference curve with elliptical profile (fig. 20).

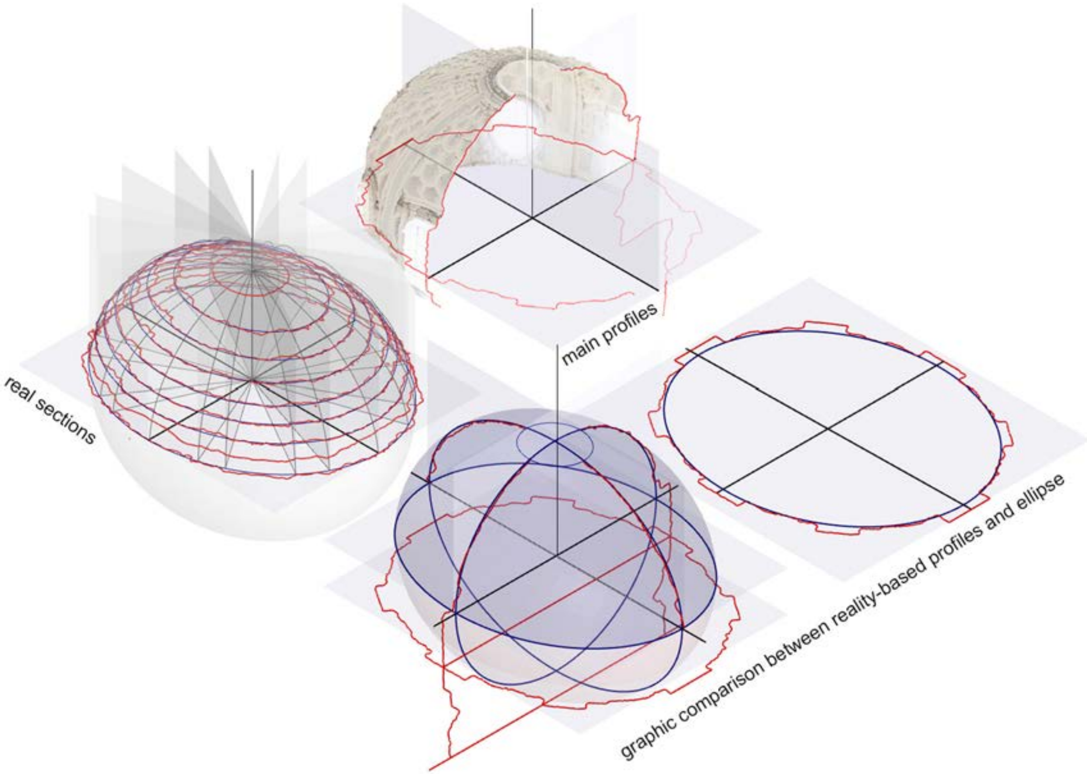

Fig. 19 - Contour of reality based textured mesh of the dome intrados of Padri della Missione church (horizontal and radial vertical sections) and modelling of revolution ellipsoidal shape. Comparison between reality based and 'semi-ideal' models.

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9. CONCLUSIONS AND FUTURE WORKS

The applications show that GAs are useful tools for geometric interpretation and graphic restitution of complex shapes (curves or surfaces). We are going to deepen accuracy of these models and improve current definitions. Future advances could be in testing other kinds of algorithms (Gatti, 2020), [3]. Semi-automatic interpretation of complex elements simplification and their parameterization according to interoperable logics (VPL/BIM-HBIM) aimed at designing new tools and/or functionalities. Furthermore, dissemination of this geometric-speculative approach could also support studies aimed at different goals. The aim of current and future research is also to experiment the same process for other kinds of curves (such as other conics), with the aim of identifying the curve that best suits the irregular reference polyline (reality based).

NOTE

[1] Euclide, 300 a.C. ca. Elementi. Libro XI.

[2] Apollonio di Perga, fine 300 a.C. ca. Coniche, Libro I, def. 1.

[3] Ottimizzazione stocastica. Wikipedia. it.wikipedia.org/wiki/Ottimizzazione_stocastica [12.03.2021].

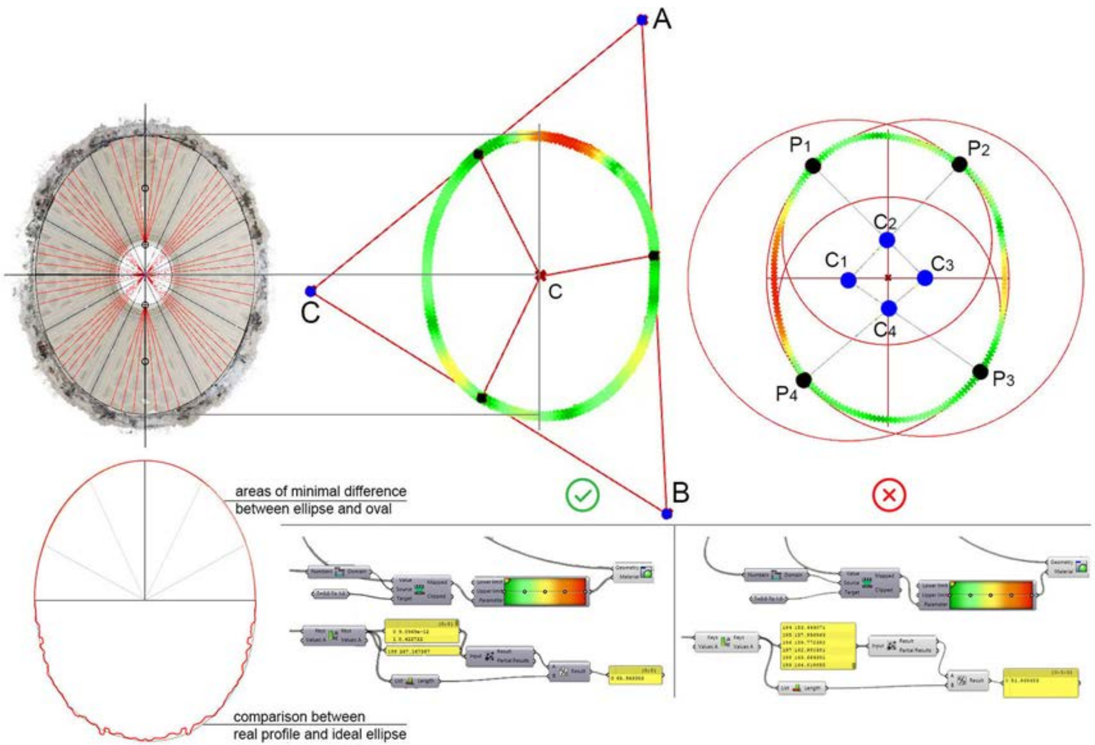

Fig. 20 - Comparison between the reality based profile of the Padri della Missione dome, elliptical and oval profiles: elliptical shapes best fits the reference curve.

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