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Elena Eramo
Researcher (ICAR/17) and lecturer in Drawing at the Department of Civil Engineering and Computer Science of the Tor Vergata University of Rome.
Her current research focuses on the digital modelling of archaeological heritage for risk assessment and for studying Roman age vaulted structures.

Filippo Fantini
Associate Professor (Drawing - ICAR/17) at the Department of Architecture of the University of Bologna. Ph.D. in Sciences of Survey and Representation at University of Florence.
Current projects focus on design analysis of vaulted spaces carried out with digital survey, 3D modeling, procedural and reverse modelling techniques.

An integrated approach for investigating roman cupolas: from segmented models to *trikentron* analysis

Segmental vaults built in the 2nd century A.D. represent perhaps the most distinctive characteristic of Hadrianic architecture. These composite vaults integrate sections of traditional shapes, such as spheres and cylinders, with more complex geometries, like the *trikentron*. These intricate structures pose substantial challenges in both the comprehension of their original construction and contemporary conservation efforts.
This paper focuses specifically on Hadrianic vaulted systems based on circular plan layouts, such as the Serapeum at Hadrian's Villa, the Horti Sallustiani in Rome, and the Temple of Venus in Baiae, by analysing reality-based mesh models generated from laser scanning and photogrammetry data. Leveraging advanced digital tools, including automated shape recognition, best-fit algorithms, and VPL-based modelling environments, the research aims to deepen our understanding of the geometrical principles underlying the design of these structures. A central objective is to correlate these findings with ancient architectural knowledge, providing insights into the design methodology of the time. This integrated approach highlights innovative aspects of Hadrianic architecture and explores the geometrical constructions employed by Hadrianic architects to achieve these complex designs.
The study confirmed the pioneering use of ellipse-based geometries in the Serapeum at Hadrian's Villa, challenging conventional interpretations of Roman dome construction. Additionally, the proposed workflow has proven effective in abstracting features of incomplete forms, which were then utilised in further geometric analyses. Notably, this research sheds new light on the design of the *trikentron*, a key innovation of Hadrianic architecture, identifying a common design approach across different instances of its use.

Keywords:
Design analysis; Segmentation; Reverse Modelling; Roman Domes; Heron of Alexandria

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INTRODUCTION

Automatic analysis and segmentation of reality-based mesh models to obtain mathematical representations (based on NURBS or solids) has been a standard in industrial design and mechanical and aerospace engineering for years (Gaiani, 2006). The techniques used in these fields can provide new and more in-depth keys to interpreting buildings, even well-known ones which underwent several studies, using an integrated analysis workflow (Cipriani et al., 2017a). In recent years, this approach has become increasingly widespread in archaeology, leveraging various segmentation approaches, sometimes based on machine learning methods. Such techniques are particularly suited to the archaeological domain due to the need to interpret objects that are often only partially preserved and generally degraded by diverse factors. However, research advancements in this domain primarily focus on replacing manual and time-consuming massive documentation and analysis of small artefacts or fragments of decoration (Jiménez-Badillo et al., 2010; Tal, 2014; Di Angelo et al., 2022) or on the analysis of large datasets at a landscape scale (Davis, 2019), mainly acquired through airborne and satellite sources. Conversely, applications focused on architectural and geometrical analysis of archaeological complexes remain a relatively less explored area, though reality-based mesh models are nowadays a standard for representing survey information.

Segmental domes developed from the second half of the 1st century A.D. onwards can be considered emblematic and challenging case studies from the perspective of design, building logistics and, not least, current conservation. These shapes, spanning from the principate of Nero to that of Hadrian, reach peaks of morphological complexity and are the product of eminent personalities of ancient architecture and engineering (Conti & Martines, 2010). Nevertheless, over time, several deterioration phenomena and anthropic alterations arose, in addition to restorations not adhering to the original drawing, complicating the understanding of their original design.

Fig. 1 - Segmental Hadrianic vaults on circular layouts: (top) Horti Sallustiani, Rome; (bottom) Serapeum, Hadrian's Villa, Tivoli; (right) Temple of Venus, Baiae, Bacoli.

Under Hadrian, there was a flourishing of such composite domes, characterised by the alternation of segments that can be categorised into two types: sections of regular geometric shapes, typically cylinders and spheres, henceforth referred to as "coves", and triangular "sail vaults", termed *trikentron* by Heron of Alexandria (Roca et al., 2023). *Opus caementicium* was the building material that made the construction of such plastic domes possible through skilful and expensive centring works, which allowed this sort of "artificial stone" to be moulded into various shapes. Given the innovative features of the Hadrianic domes and their limited success in immediately subsequent times, it is problematic to strictly apply a method of analysis based on the typological comparison and protocols widely spread in the archaeological field.

Hadrianic domes, i.e., those attributable to the imperial will for formal experimentation, survive in an exiguous number, thus limiting the possibility of making extensive comparisons [1]. Even geographically, this production is confined to Rome, Tivoli, and Baiae, and only a few antecedent cases can be related to it (mainly Roman and Baian examples from the Neronian and Domitian ages (Cipriani et al., 2017b)). Similarly, a small number of relatable examples, immediately following the Hadrianic principate, can be found in thermal bath architecture (octagonal halls at the Baths of

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Diocletian in Rome, the Baths of Bacuccus in Viterbo, a thermal complex in Otricolum) and testify to the permanence of only a few formal elements derived from Hadrianic models. Later receptions may be found in other isolated cases, showing interesting reworking of themes already implicit in the Hadrianic-centric spatialities, as in the Church of Saints Sergius and Bacchus in Constantinople (Mango, 1976). However, they will not find as much fortune compared to morphologically simpler and undoubtedly cheaper solutions in terms of construction logistics and materials used. A primary example of these is the basilica of San Vitale in Ravenna, where we observe a reinterpretation of planimetric themes close to those of Hadrian. Nevertheless, its thin hemispherical dome in fictile tubes is enclosed by a *tiburium* that cannot be traced back to the known experiences of the 2nd century A.D.
Therefore, the exceptional nature of the "parenthesis" constituted by the Hadrianic experiments calls for an innovative approach that is both integrated and open to revising many now-historical beliefs, often lacking adequate graphic documentation to support the keys provided about the design of vaulted systems.
Let us briefly analyse the characteristics of Hadrian's production of vaulted systems, utilising their plan layout design (*ichnographia*) as the primary criterion for typological subdivision. A first group presents octagonal layouts. Several examples from Hadrian's Villa belong to this category: the irregular octagonal room with niches, with a distributive function, of the Baths with Heliocaminus (Cipriani et al., 2017b); the mixtilinear octagonal hall of the Small Baths (MacDonald & Boyle, 1980; Mollo & Marzuoli, 2020) and the vestibule of the Golden Court (Giuliani, 1975) similarly based on octagonal patterns [2]; the room with a mixtilinear layout above the tower of the Greek Library (Kleibrink, 2000), though in a far inferior state of preservation than the previously mentioned examples. At Baiae, we have the small building consisting of a main hall with convex sides forming cusps in the corners, surrounded by subordinate volumes (also domed), leaning against the main body of the Hadrianic building known as the Temple of Venus (Rakob, 1988). A second group is instead set on circular layouts: the Serapeum at Hadrian's Villa, which presents a segmental dome on a semicircular drum (Eramo, 2023), the circular hall at the Horti Sallustiani (Moretti, 2000), in Rome and finally in Baiae – the larger example of this typology – the Temple of Venus (Rakob, 1988) (Fig. 1).
In the listed vaults, we can also identify two somehow contrasting trends in the connection and alternation of convex and concave surfaces, leading to different geometric shapes: "tents" and "shells", or gourds, following Apollodorus of Damascus derogatory definition (Brown, 1964). Emblematic examples of these typologies, respectively, are the octagonal hall of Small Baths and the circular hall of Horti Sallustiani. The former, perceptually elusive, features a composition of four concave *trikentron* (Roca et al., 2024) alternating with cylindrical sectors, resembling an evoluted version of cloister vaults typology. The latter shows a circular arrangement, where the alternation of surfaces is particularly emphasised: each spherical triangular sector is embedded between two *trikentron*.

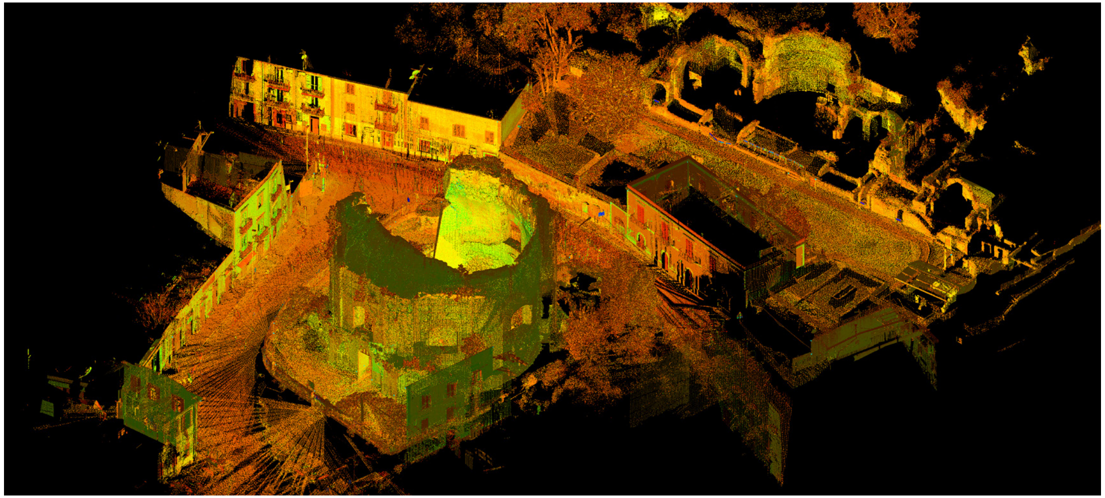
Fig. 2 - Point cloud data from the laser scanning survey campaign of the Temple of Venus and upper terrace in Baiae, Bacoli, February 2024.

AIMS

This study aims to clarify the geometric nature of the latter typology by developing a methodology that integrates tools for the automated recognition of geometric primitives. Notably, the paper examines the geometric shapes underlying the design of the coves and *trikentron*, investigating whether these shapes are traditional primitive geometries or more elaborated forms resulting from sets of curves - generatrices and directrices - consistent with the complex centring work required for constructing the domes. Simultaneously, to ensure that the results contribute to the effective understanding of the original design, a crucial objective is to develop a method that integrates the analytical research dimension - based on sensors and geometric shape recognition systems - with an approach based on knowledge from ancient treatises and architectural manuals. Reconciling the quantitative approach with insights from geometric constructions found in ancient written sources presents a challenging problem, requiring sensitivity and expertise in both areas.
Finally, building on the results of recent research,

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this work particularly highlights the new elements that have emerged on the Serapeum, arguably the most representative building of Hadrianic architecture, whose segmental vault has always fascinated scholars from various disciplines and times.

DATASET AND SHAPE ANALYSIS

The surveys conducted on the centric buildings characterised by domes and complex vaulted systems at Hadrian's Villa are the results of diverse campaigns carried out by the University of Bologna in collaboration with the Istituto Villa Adriana and Villa d'Este. The combined use of active and passive sensors enabled the production of digital representations of the complex and damaged shapes with an average resolution below 1cm. The laser equipment utilised included the Leica Scan Station C5, based on ToF technology (Benedetti et al., 2010), provided by the Department of Architecture of the University of Bologna, and the Zoller+ Fröhlich IMAGER 5010C, which employs Phase Shift technology [3]. The photogrammetric campaigns for the Serapeum, the circular hall of the Horti Sallustiani, and the Temple of Venus in Baia were conducted employing a Single Lens Reflex Camera Nikon D5200, equipped with a Nikon AF-P 18-55mm f.3.5-5.6 DX VR lens and an X-rite ColorChecker reference colour target. Data processing from laser scanning was performed through Leica Cyclone 9, while the meshing and reverse modelling were carried out with 3D System Geomagic Design X (Cipriani et al., 2017a; Eramo & Cinque, 2024). The survey campaign for the Temple of Venus in Baia, conducted in collaboration with the Campi Flegrei Archaeological Park and Tor Vergata University of Rome, took place in January and February 2024, utilising the Leica Scan Station C5 equipment (Fig. 2).

To analyse the shape of alternated coves and *trikentron* forming the intrados of the surveyed domes, an initial simplifying hypothesis was employed regarding the compositional hierarchy of the two types of segments. This hypothesis posits that the design of the *trikentron* was subordinate to a general shape design, which can be traced back to a single geometric primitive, e.g. a sphere, and aligns with the shape of the coves. Therefore, the portions of the model corresponding to the coves and the *trikentron* were first manually segment-

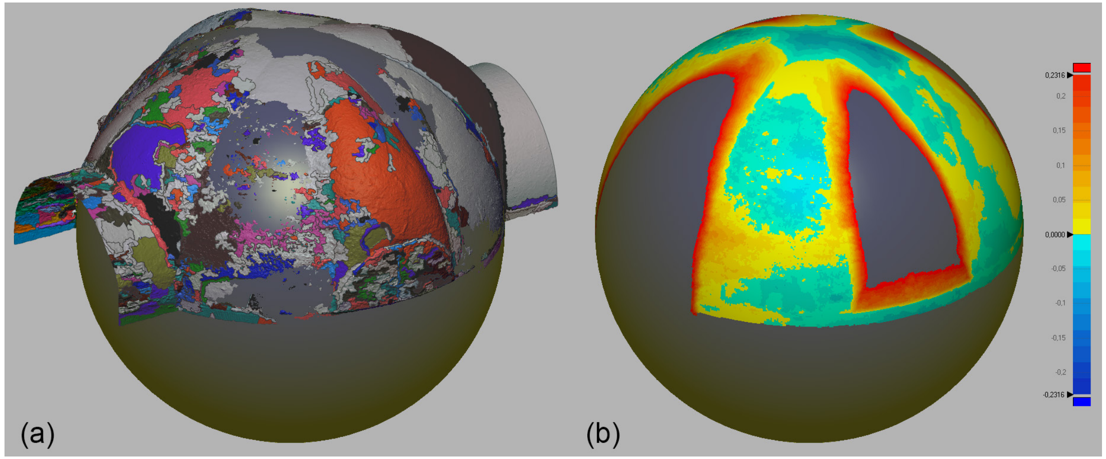
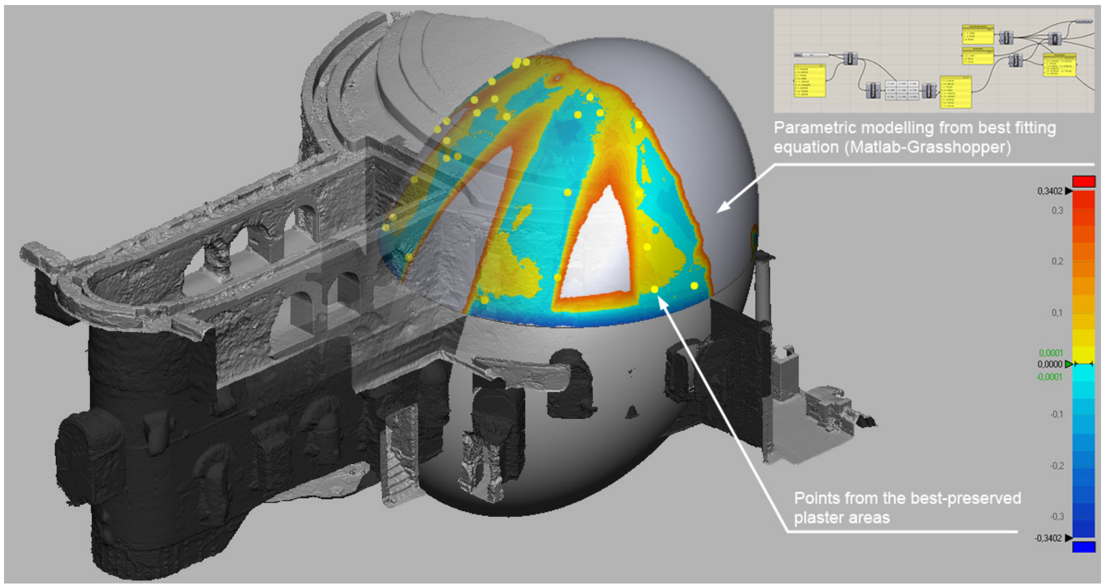


Fig. 3 - (a) The best-fitting sphere for the coves of Horti Sallustiani mesh model. (b) Colour map of the mesh deviation between the sphere and the mesh model.

Fig. 4 - The complex segmental vault of the Serapeum did not fall within the standard primitives of shape recognition tools and required the development of an *ad hoc* methodology.



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ed. Then, for each segment, homogeneous areas were identified based on the conservation state of the surface: original casting surface, presence of plaster coatings (often showing considerable thickness up to 15cm), and highly degraded areas. This distinction ensured that subsequent analyses could provide answers consistent with the original design.

Subsequently, the sets thus prepared were analysed utilising geometric-based segmentation tools already integrated into commercial software, notably Geomagic Design X, widely regarded as an industry standard for reverse modelling. Concurrently, the same sections underwent analysis in the open-source software Cloud Compare, utilising the RANSAC Shape Detection tool that leverages the algorithm developed by Schnabel et al. (2007). Though these tools employ slightly different recognition methods, they both entail identifying a limited set of geometric primitives, including planes, cones, spheres, cylinders, and tori This approach allowed for the development of a semi-automated workflow suitable to recognise spheres and cylinders, common shapes employed in the design of segmental vaults, as in the case of Horti Sallustiani (Fig. 3). However, this set of primitives was not sufficient in resolving the fitting issues encountered in the more complex case of the Serapeum.

Building on the earlier identification of an ellipsoid-based shape (Eramo & Cinque, 2024), a dedicated workflow was devised to fit the models with quadric surfaces. Identifying the best-fitting ellipsoid for 3D datasets is a thoroughly explored topic in computer vision, and robust algorithms tested across various applications, from medicine to game design, are available. Therefore, the fitting was performed through a Matlab function (Petrov, 2015) that approximates a set of points with the optimal fitting quadric surface (ellipsoid, sphere, paraboloid, hyperboloid) via least square optimisation devoid of specific initial constraints. Subsequently, the results were imported in a Rhinoceros-Grasshopper VPL code, facilitating the recreation of ellipsoidal mesh surfaces by employing non-uniform deformation and rigid rotation of

MESH DEVIATION RESULTS

SET NAME	Min [m]	Max [m]	Average [m]	St. Dev. [m]	Int. ±1σ [Poly-Vertices %]	Int. ±2σ [Poly-Vertices %]
Coves - casting srf	-0.1274	0.2750	0.0002	0.0174	75.8959	96.3831
Coves - global	-0.5740	0.5921	0.0029	0.0665	74.4983	95.5531
East trikentron - casting srf	-0.0729	0.2600	0.0005	0.0257	75.3717	95.0271
East trikentron - global	-0.1367	0.4496	0.0022	0.0530	79.8578	95.7251
Central trikentron - casting srf	-0.1116	0.4600	0.0007	0.0486	76.7849	95.9956
Central trikentron - global	-0.1644	0.5900	0.0108	0.0740	86.0397	96.9180
West trikentron - casting srf	-0.1317	0.4545	0.0007	0.0349	76.8310	95.8662
West trikentron - global	-0.1317	0.5322	0.0240	0.0751	81.6750	94.7461

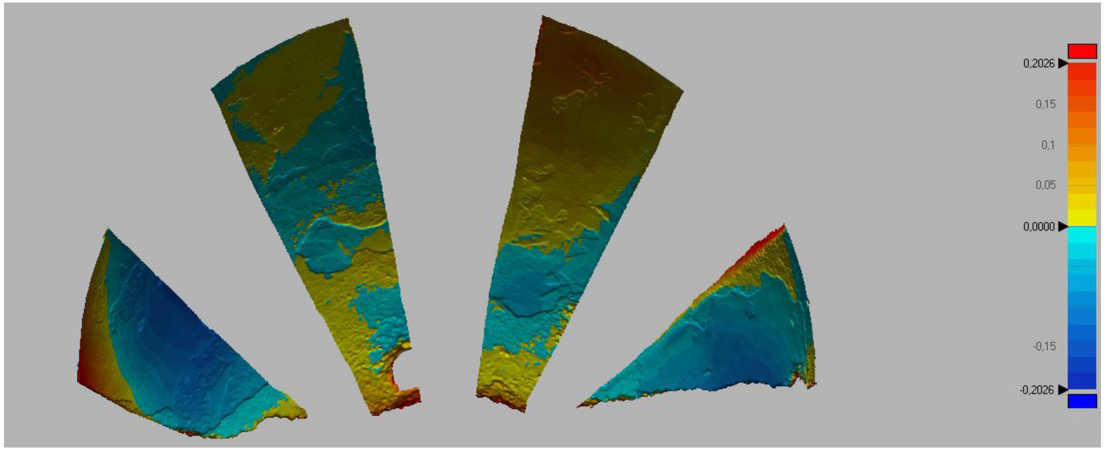

Fig. 5 - (a) Table summarising the mesh deviation analysis results for the assessed sets. (b) Colour map displaying the mesh deviation analysis for the "Coves – global" set.

an initial generic sphere (Fig. 4). This methodology enabled the evaluation of fitting results through mesh deviation analysis in Geomagic Design X. At first, the ellipsoids were compared with the segmented casting surface sets, and then with the entire corresponding segment set, encompassing plaster and degraded areas.

Four segments of the model were thus analysed: the east, central, and west *trikentron*, as well as the four coves collectively. For each segment, coordinates of points belonging to the original casting surface were extracted from the mesh models (with a grid spacing of 0.5 mm), ordered and divided into ten independent sub-sets. The resulting quadrics from the fitting process consistently yielded ellipsoids. The fitting results for each subset (centre, radii, eigenvectors) were then assessed for each segment, deciding to utilise their average values due to absolute errors remaining below 2 cm.

The table in Fig. 5 summarises the successive mesh deviation analysis results. Most of the analysed mesh model surfaces fall within approximately 13 cm from the fitted ellipsoids for the

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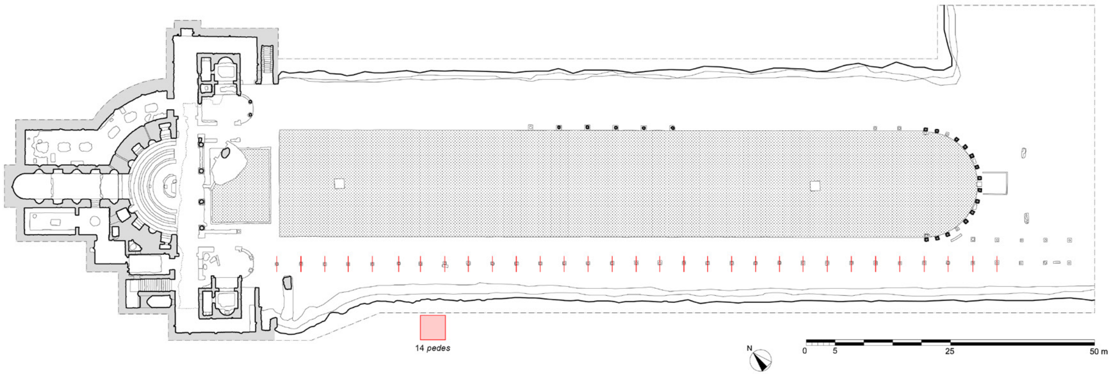
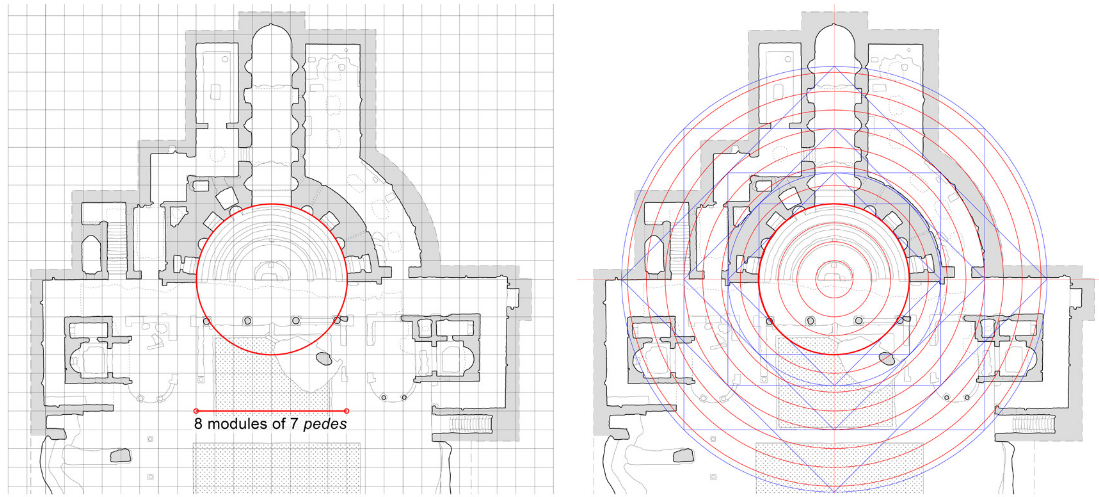


coves and approximately 15 cm, at worst, for the *trikentron* (Fig. 5a). Indeed, interpreting these deviation values requires observing different aspects concerning the ellipsoids fitted for the coves and *trikentron*. Regarding the coves, while error values are not negligible in absolute terms, it is necessary to consider the conservation state of the vault. In this case, the error distribution is consistent with the presence of thicker layers of plaster or areas that are more heavily degraded (Fig. 5b). Moreover, the resulting surface exhibits an overall shape that aligns with the simplified analyses and qualitative interpretations developed in previous studies. The geometric analyses detailed in the following section further support this strong correspondence. Concerning the *trikentron*, regardless of the error measurement, it is noteworthy to note a significant deviation between the theoretical surface and the reality-based model. This deviation is particularly evident in the impost area near the connection with the lunettes and the summit area towards the top of the covering, where the surface should be tangential to the main ellipsoid. Hence, while the initial best-fit ellipsoid appears to serve as the foundational geometric primitive for the overall geometry of the vault, this shape is insufficient to explain the formal genesis of the *trikentron* elements. Nonetheless, this analysis has revealed, through the ellipsoidal surfaces, directional aspects in their development and indicative positions of their symmetry axes.

Fig. 6 - Plan of the Serapeum-Canopus complex with the 14 pedes modular grid.

Fig. 7 - (a) Modular grid of 7 pedes overlayed on the Serapeum plan. (b) The *ad quadratum* scheme that rules the dimensions of the complex.



(a) (b)

4. DESIGN ANALYSIS

The results of the shape analysis, as previously stated in the aims of this work, are intended to help understand the original design of these shapes. Therefore, they must be interpreted in the context of the knowledge available to ancient architects. In continuity with previous studies of vaulted buildings, we aim to retrace the sequence of steps, calculations and graphic schemes (*dispositio*) performed by the ancient architects (Adembri et al., 2015).
In the Serapeum-Canopus complex, a modular grid (Gonizzi Barsanti et al., 2022) is initially recognisable, emerging from the analysis of the interaxis of the sequence of columns on the east side of the great Canopus basin. The module value, based on the average of the measurements, is 14 pedes (Fig. 6). It is worth noting that the diameter of the large semicircular hall housing the *stibadium*, measured at the impost height of the semi-dome, is equal to 4 modules of 14 pedes. The use of a multiple of the number 7 in such centric spaces is a recurring feature in imperial Roman architecture. The modularity can be attributed to

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the approximation of π to 22/7, as found in Alexandrian-derived formularies attributed to Heron of Alexandria (Juan-Vidal et al., 2024). The modular grid and the use of the number 7 also coexist with the ad quadratum scheme, which governs the dimensions of the spaces surrounding the hall and the elements within the room covered by the extensive segmental dome (Fig. 7).
Regarding the design of the vaulted system, the overall shape of the Serapeum cannot be identified as a sphere, unlike the Horti Sallustiani, whose spherical design has been verified, and, most likely, the Temple of Venus. However, for the Serapeum, the developed methodology confirmed the hypothesis proposed in recent studies regarding its ellipsoidal arrangement. Its dome has always been interpreted as spherical in literature. More generally, the use of ellipsoidal shapes (or their oval approximations) for domes and vaults is documented as characteristic of modern-age architectures, or explicitly considered an exception limited to certain peculiar cases from the Roman age (Huerta, 2007). The results discussed here, therefore, would define a novelty in the field of construction history, albeit leaving several questions open regarding its design and construction procedures, which are currently under investigation.
To delve into the original conceptual process behind such an extraordinary form, it is essential to explore the mathematical and geometrical theories available during that era. Notably, Archimedes' renowned treatise "On Conoids and Spheroids" provides comprehensive information for tracing sections and computing volumes of three-dimensional spheroidal shapes, namely ellipsoids generated by rotating an ellipse around one of its axes, with two principal radii of the same measure (Frajese, 1974). However, in the case of the Serapeum, despite the uncertainty associated with the discussed metric errors, the investigated form undoubtedly embodies a generic ellipsoid with three distinct radii, particularly notable for having the major axis inclined towards the direction of the complex's facade.
If, despite the lack of direct evidence, we speculate that such theories had already been extended to the more general case at the time of Hadrian, currently, the most plausible hypothesis is that the project was conceived according to a "deformative" principle. The idea stems from reversing the reasoning behind the initial hypothesis. This hypothesis was developed by drawing the best-fitting circles of planar sections and recognising a sort of shear deformation, in other words, an affine transformation, applied to a spherical geometry with horizontal tangents coinciding with those of the ellipsoid. This deformation was identified based on the alignment of the centres of these circles along an inclined axis (Fig. 8) A shape akin to the one identified in the analysis of the Serapeum mesh model could have been conven-

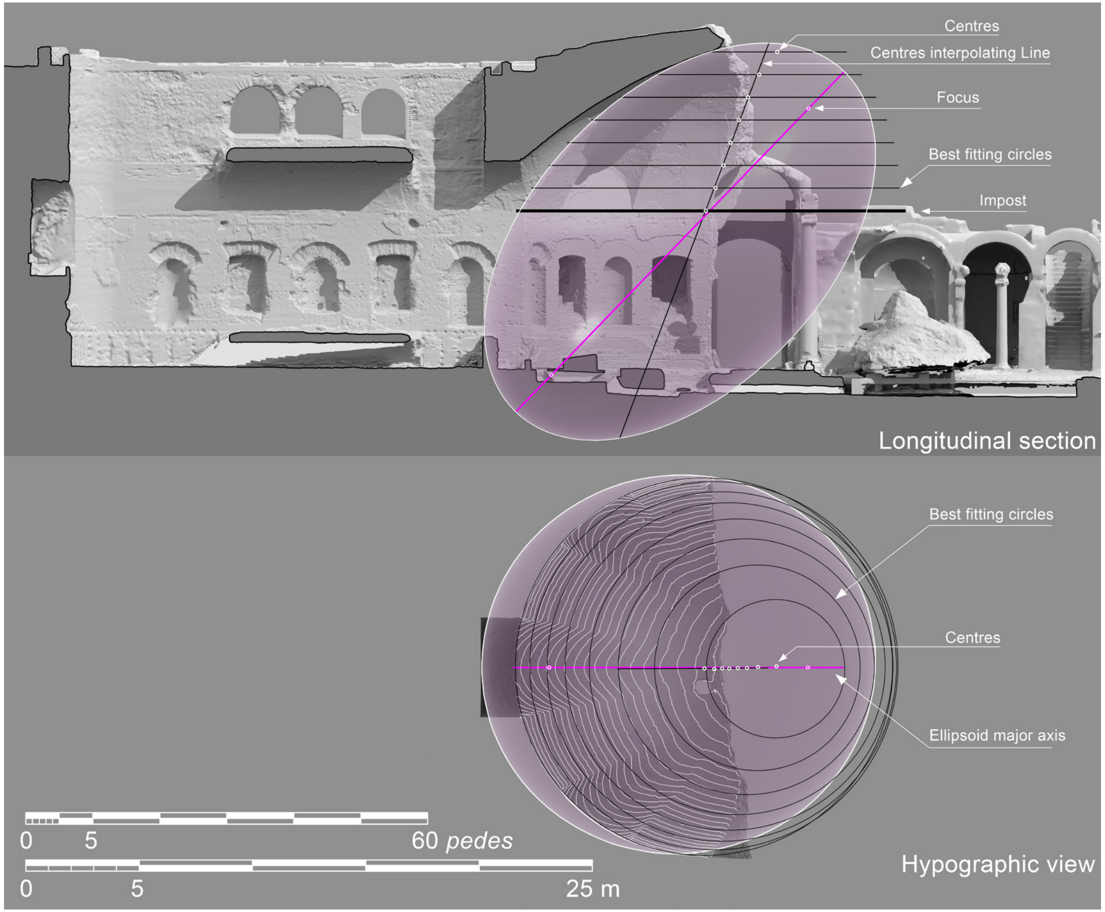

Fig. 8 - Longitudinal section and hypographic view of the mesh model of the Serapeum, with the results of the simplified analysis (best-fitting circles), the best-fitting ellipsoid and the focii of the ellipse resulting from its longitudinal section.

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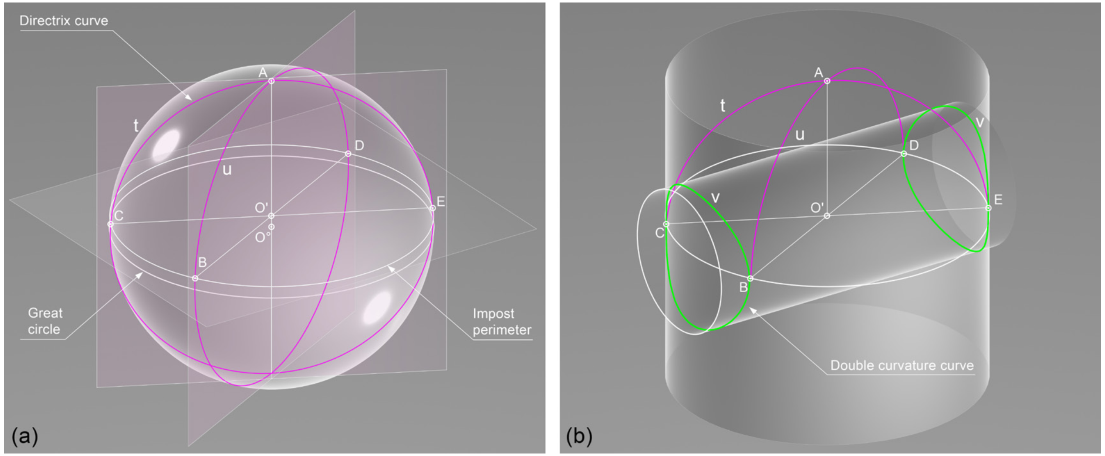

Fig. 9 - Horti Sallustiani: (a) primitive sphere intersected with vertical planes. (b) Double curvature curve v resulting from the intersection of two cylinders.

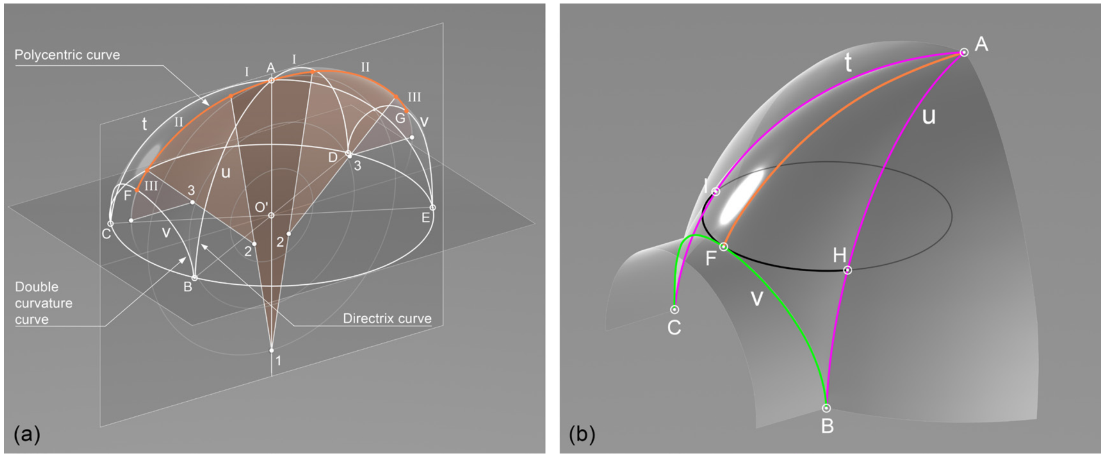

Fig. 10 - Horti Sallustiani: (a) the central director arc is a polycentric curve. (b) The three director arcs and horizontal circular profile.

iently constructed by translating circular curves, "point by point", on the lateral and frontal elevations drawings, adhering to a rule determined by a rotated axis. The development of an accurate tracing methodology for the shape identified in the Serapeum remains an ongoing investigation. Nevertheless, similar considerations have proven valuable for analysing the design of the *trikentron*.

TRIKENTRON ANALYSIS

The three Hadrianic buildings that are most representative of the "shell" typology, in addition to having a circular or semicircular plan, present alternating *trikentron* and coves; as already stated, these latter derived from solid primitives such as spheres or ellipsoids. The lunettes obtained along circular-shaped layouts are double curvature curves (not lying on a plane) resulting from the intersection of the vertical-axis cylinder obtained by extruding the base circle with horizontal-axis cylinders. The intrados surface (spherical or ellipsoidal) is intersected by planes (often vertical) that define two director arcs for each *trikentron* (Fig. 9).

The analysed *trikentron* appear to be generated similarly but with significant differences depending on the spherical or ellipsoidal nature of the primary starting surface. In general, however, it is observable that such surfaces are the product of three director arcs along which circular profiles with decreasing diameters from the lunette to the top of the vaulted system. In the case of the Horti Sallustiani, the level of conservation is much higher than that of the Serapeum. Thus, it allows us to extract more reliable contour lines than the Tiburtine scenographic *triclinium*. This one partly collapsed, in typological terms, is much more complex since it lacks polar symmetry: the eight segments of the Horti Sallustiani, organised along a circular outline, correspond to 9 segments arranged along a semi-circle that juts out in the direction of the Canopus' euripus. However, the central director arcs of the *trikentron* of the two buildings - spines along which to pass circular arches - share some common features. Geometrically, the central arches of the Horti Sallustiani are easily recognised as polycentric curves by the reverse modelling application adopted (Fig. 10). The curve appears continuous, indicative of high construction expertise resulting from the architect's geometric mastery; however, the application is not able to achieve a full interpretation of how the three circles "chain", in continuity of tangency, leads to the final shape of the central director arc. An interpretation was then proposed that led us to a possible construction in line with what has already been proposed in the existing literature related to the ichnographic tracing of amphitheatres (Migliari, 1995).

This approach was applied to the two longitudinal *trikentron* (aligned to the access to the circular hall) and the two transverse *trikentron*, which

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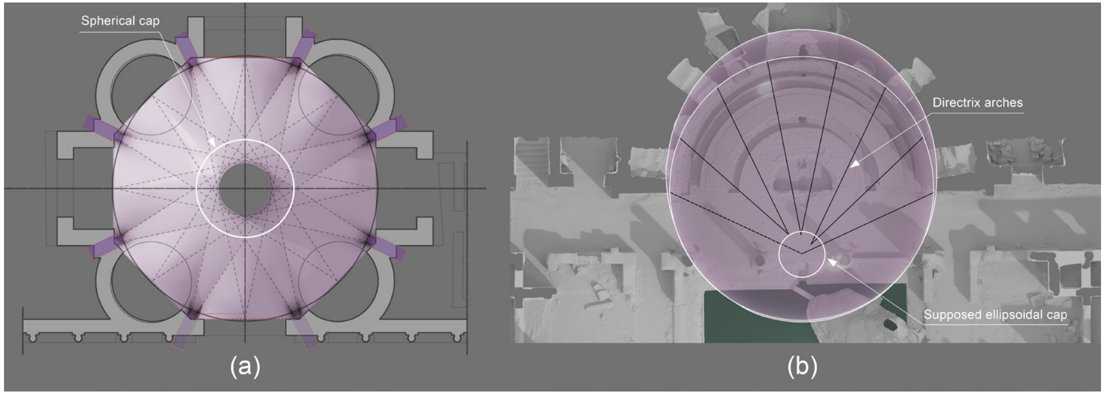


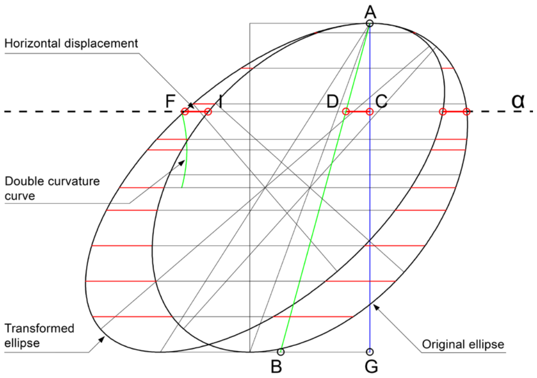
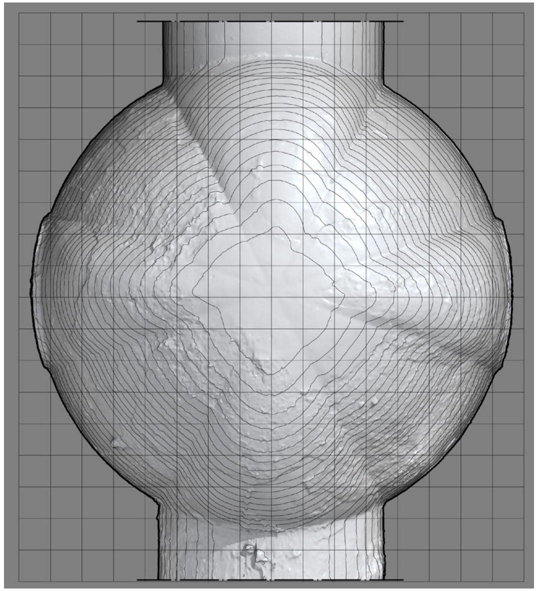


Fig. 11 - (a) Golden Court vestibule: upper spherical cap. (b) Serapeum supposed upper ellipsoidal cap.

Fig. 12 - Scheme for determining the transformation of the ellipse derived from the ellipsoid into the directrix curve of the central *trikentron*.

Fig. 13 - Contour lines diagram of the vault of Horti Sallustiani.



have different boundary conditions because of the lower side lunettes. Both pairs of curves, however, have in common the condition of presenting a stationary point at the dome top; in other words, it means that the base sphere, at its upper pole, is tangent to both central director arcs.
The formal analysis of the analogous *trikentron* in the Serapeum is not as clear. There are essentially three reasons for this:
1. all the *trikentron* are incomplete, especially in the upper area of the vaulted system.
2. two of them are substantially collapsed, except for a limited stump preserved west of the dome, but still not sufficient for an unequivocal interpretation.
3. the central director arches present a very low curvature that, when interpreted through best-fitting circles within computer applications. Best-fitting circles lead to solutions adherent to the model but geometrically disconnected from the overall context of the building as well as from the technical-scientific knowledge of the ancient architect.
The access vestibule to the Golden Court of the Hadrian's Villa, subject of previous studies (Cipriani et al., 2020), is also characterised by the presence of *trikentron*. It appears that the lateral director arches of the *trikentron* did not have a common point at the upper pole of this spherical dome. If the *trikentron* do not meet at the highest pole of the sphere (Golden Court vestibule) or ellipsoid (Serapeum), it means that there is a spherical or ellipsoidal cap around the top of the dome (not interested by concave gores). However, in the case of the Serapeum, the interpretation of the overall system of directrices is not straightforward (Fig. 11), and the closer we get to the collapsed concave wedges, the less intuitable their shape is.
Aware of the inevitable aporia arising from such gaps, which moreover cannot be solved through the known iconographic sources, it is nevertheless possible to analyse the best-preserved *trikentron*, namely the one axial to the complex, symmetrical with respect to the longitudinal plane passing through the middle of the building. Briefly retracing the steps previously listed, we get (through the intersection of two cylinders) a double curvature curve, sliced on a barrel vault leading to the final corridor of the Serapeum. Then, two elliptical directrices, obtained by sectioning the basis ellipsoid, meet at a point belonging to the boundary of the upper cap of the dome (Fig. 11 b). Since the general interpretation of the vaulted system of the building is based on the concept of deformation by horizontal displacement of sections belonging to a sphere, we assume a similar procedure to interpret the criterion employed by architects to draw the central director arc of the *trikentron*.
It is assumed, therefore, that the ancient architect used an elementary geometric construction that was easy to put into practice, even logistically. The director arc was defined by two requirements: the

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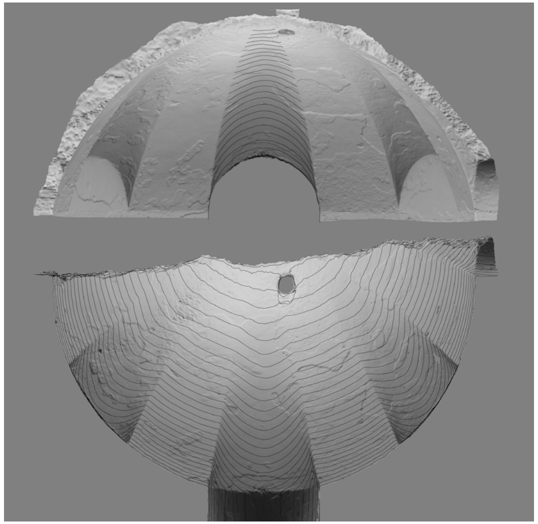
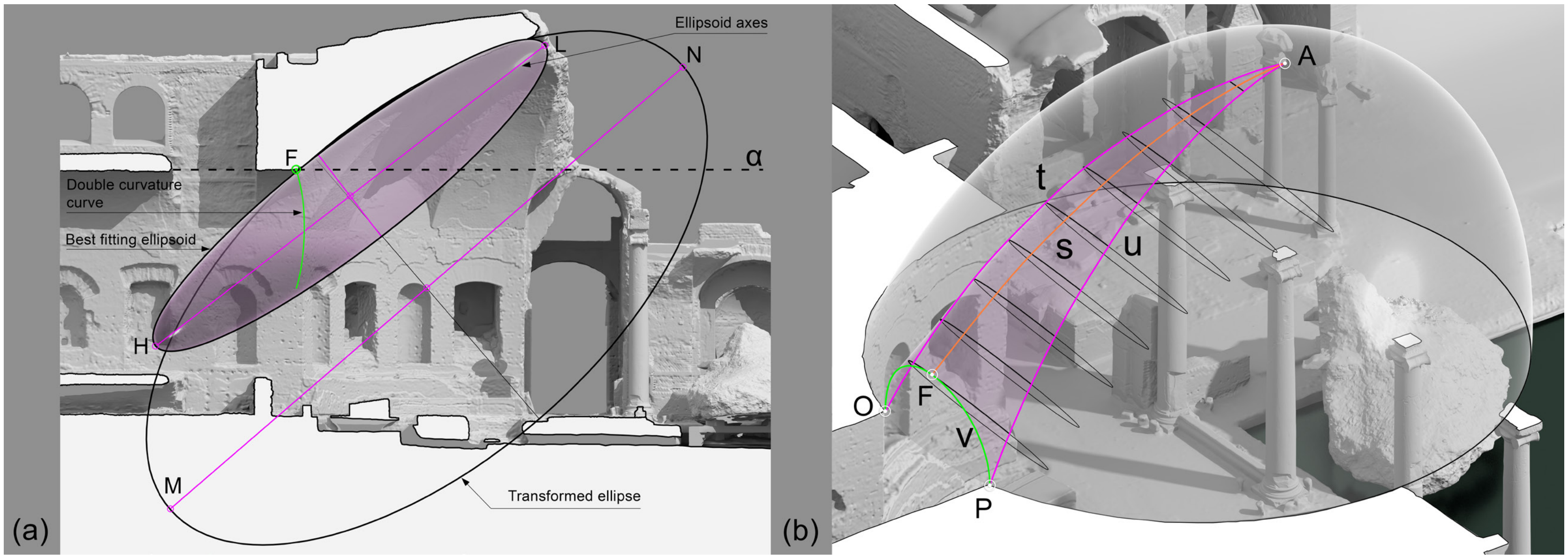


Figure 14. Contour lines diagram and vertical sections of the central *trikentron* of the Serapeum.

first is the constraint of tangency with the upper point A of the lunette, and the second is that the lower end of that curve would match the top of the double curvature curve v, i.e., point F (in Fig. 12). We can think of the central director arc of the *trikentron* as a deformation of the longitudinal section of the ellipsoid. The construction is simple since all that is needed is to identify a horizontal displacement quantity from which all others descend. This quantity is the segment FI (Fig. 12) belonging both to the horizontal plane α, passing through F, the point of maximum of the curve v, and to I belonging to the longitudinal section of the ellipsoid. By horizontally displacing the segment FI to the segment perpendicular to it AG, we determine the point D. The segment AB passing through D defines the "rule" to trace the horizontal displacement of each horizontal segment belonging to the starting ellipse. In the case of the Horti Sallustiani, the contour lines of the *trikentron* make it clear that the generatrices are parallel to the horizontal plane and consist of circles of decreasing radii (Fig. 13).
In the *trikentron* of the Serapeum, the same condition does not arise: the contour lines are blatantly different from circular profiles, and even the tests carried out by sectioning the *trikentron* with vertical planes do not allow the detection of circular profiles in a clear manner (Fig. 14).
This issue is crucial to drawing a reliable hypothesis on the geometric construction of these concave gores, given the close relationship between generatrix curves and the ribs system underlying the construction of formwork suitable for performing *opus caementicium* vaults. We observe that the best-fitting ellipsoid on the central *triken-*

Fig. 15 - (a) Longitudinal section of the Serapeum showing the best-fitting ellipsoid approximating the *trikentron* and the ellipse obtained by deformation according to the described procedure. (b) Geometric construction hypothesis of the central *trikentron*: skew curve (v), lateral director arcs (t, u) and central director arc.



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tron has radii of 1.943 m, 2.148 m and 9.230 m, respectively (Fig. 15a). The major axis HL is also very close to being parallel to the maximum principal diameter of the ellipse MN that acts as the directrix arc of the central *trikentron* (in accordance with the diagram in figure 12). The other two radii appear within 2 m, and we can think of the generatrixes of the *trikentron* as being parallel, but lying on a parallel sheaf of planes whose normal is the maximum diameter of the ellipse that serves as the central directrix (Fig. 15b). Therefore, the *trikentron* would result from three director arcs, all three of which are elliptical (s, t, u), while the transverse profiles could be circular, with variable radius and parallel to each other. The result of this set of assumptions was compared with the high detail mesh and provided encouraging results as it returned an average deviation of -0.117cm (Fig. 16). This measurement, taking into account the altered state of the intrados, is quite acceptable taking into account the presence of plaster patches of the thickness of 10 and more centimetres that characterise the selected *trikentron*.

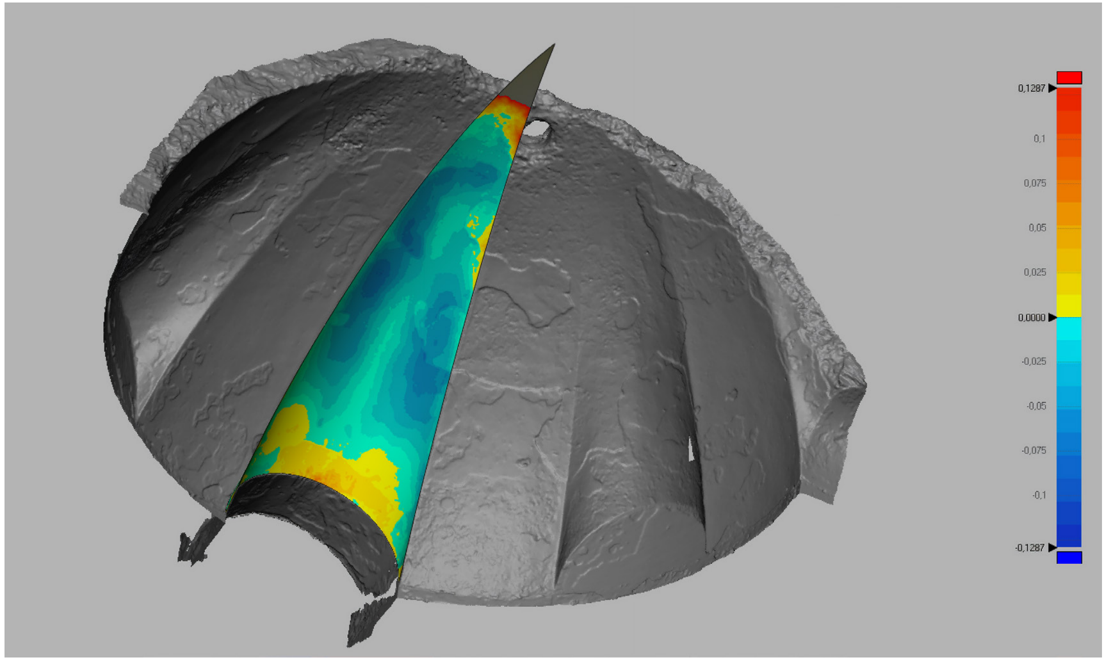

Figure 16. Mesh deviation analysis between the idealised model of the central *trikentron* and the mesh model of the vault of the Serapeum.

CONCLUSIONS

In this study, we developed a workflow to enlighten the geometric features of Hadrianic segmental vaults on circular layout, supported by the integration of automated shape recognition tools for the analysis of reality-based mesh models. Despite the limited number of case studies, the adopted methodology allowed for a deeper understanding of their shapes, making them consistent with an overall interpretation of the design of the buildings, following the Vitruvian concept of *symmetria*. The customised shape recognition workflow first corroborated the previous hypothesis of an ellipsoidal geometry for the vault of the Serapeum complex, which appears to represent a unique occurrence in Roman architecture. Secondly, for shapes that did not perfectly adhere to the considered primitives, the workflow proved valuable in abstracting features of elusive or partial forms, whose evidence was utilised in subsequent geometric analyses. Remarkably, the work provided new insights into the subject of the *trikentron*, one of the main innovations of Hadrianic architecture, allowing the recognition of a common approach to their design in different cases.

AUTHOR STATEMENT

The present research is the result of the combined work of the authors across diverse contribution roles. In writing the original draft, Elena Eramo was responsible for paragraphs 2, 3, 4, and 6; Filippo Fantini for paragraphs 1 and 5. All the presented results are derived from a collaborative synthesis of their work.

NOTE

[1] Intentionally, we will not examine more common solutions of vaulted systems that fall into the categories of barrel-, cloister- and groined vaults, as well as hemispherical vaults and their derivations.

[2] The southern hall of Golden Court, as well as the mixtilinear atrium of the Accademia from Hadrian's Villa, present a much lower state of preservation and, therefore, depending on the authors, are considered to be covered by a large dome in *opus caementicium*, or not. On the topic, see Hansen et. al. (2011); Viscogliosi (2020); Ottati (2022).

[3] The device was procured in collaboration with MicroGeo company specifically for the surveying activities conducted in April 2015 on the Serapeum-Canopus complex, under the scientific coordination of Prof. Luca Cipriani. The digital acquisition of the extrados of the dome and adjoining rooms was completed by Ph.D. Silvia Bertacchi in October 2015. Technical coordination for the operational phases of the laser scanner and photogrammetric survey of the monuments was managed by Ph.D. Gianna Bertacchi and Prof. Filippo Fantini. The photogrammetric survey of the main hall and the front of the building was completed by the Authors in March 2024.

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